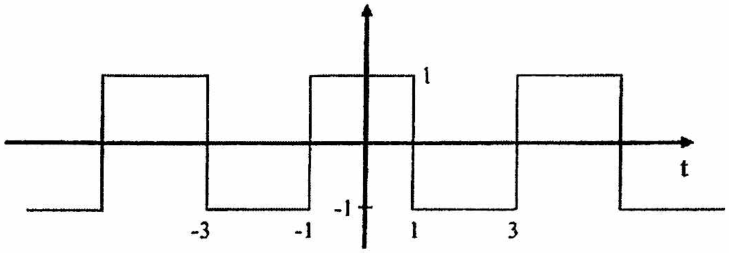
1. **Fourier Series Calculations**

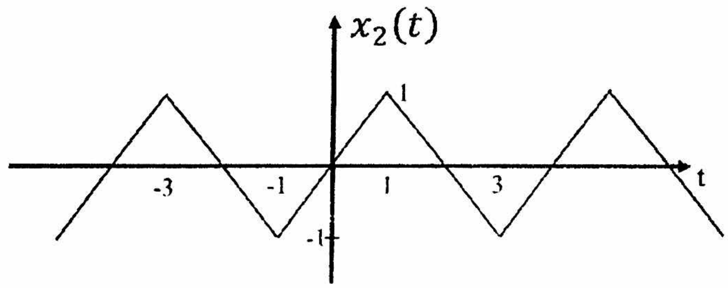
Let $x_1(t)$ be



(a) (6 points) What is an appropriate period, T_0, for this signal?

(b) (6 points) Using your T_0, set up the integral to find the Fourier series coefficients of $x_1(t)$, a_k. You do not need to solve it, but you must have the correct integrand and limits.

(c) (6 points) What would the Fourier series coefficients of $x_2(t)$ be in terms of a_k?



a) $T_0 = 4$

b) $a_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\frac{2\pi}{T_0}t} dt$

$= \frac{1}{4}\int_{-1}^{1} e^{-jk\frac{\pi}{2}t} dt + \frac{1}{4}\int_{1}^{3} (-1) e^{-jk\frac{\pi}{2}t} dt$

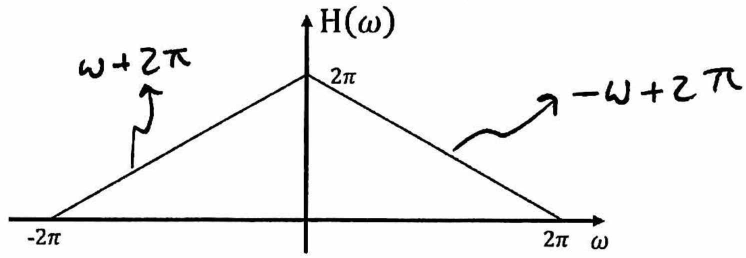
c) $x_2(t) = \int_{-\infty}^{t} x_1(\tau) d\tau \Rightarrow x_2(t) \overset{FS}{\longleftrightarrow} \frac{1}{jk(\frac{\pi}{2})} a_k$

2. **Fourier Series and LTI Systems**

Let $x(t) = 2e^{-j\frac{3\pi}{2}t} + 1e^{j\frac{\pi}{2}t} + 2e^{j\frac{3\pi}{2}t}$

(a) (10 points) Find a_k, the Fourier series coefficients of $x(t)$, using the period $T_0 = 4$. Show all your steps.

(b) (8 points) Let $H(\omega)$ be as shown below.



Find the Fourier series coefficients of $y(t) = x(t) * h(t)$, b_k.

If you did not find a_k, use $a_k = sin(k\pi/2)/(k\pi)$ and $a_0 = 1/2$ instead.

(c) (8 points) Give an expression for $y(t)$.

a) $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{\pi}{2}t}$

$= 2e^{-j\frac{3\pi}{2}t} + 1e^{j\frac{\pi}{2}t} + 2e^{j\frac{3\pi}{2}t}$

By comparing $x(t)$ with the synthesis equation:

$a_{-3} = 2$, $a_1 = 1$, $a_3 = 2$

all others are zero

b) $b_k = a_k H(\omega_0 k)$

$b_3 = a_3 H(-\frac{3\pi}{2}) = 2(-\frac{3\pi}{2} + 2\pi) = 2(\frac{\pi}{2}) = \pi$

$b_1 = a_1 H(\frac{\pi}{2}) = 1(-\frac{\pi}{2} + 2\pi) = \frac{3\pi}{2}$

$b_3 = b_3 H(\frac{3\pi}{2}) = 2(-\frac{3\pi}{2} + 2\pi) = 2(\frac{\pi}{2}) = \pi$

all other b_k are zero

c) $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{\pi}{2}t}$

$= \pi e^{-j\frac{3\pi}{2}t} + \frac{3\pi}{2} e^{j\frac{\pi}{2}t} + \pi e^{j\frac{3\pi}{2}t}$

3. **Continuous Time Fourier Transform**

(a) (6 points) Find $X_1(\omega)$ given $x_1(t) = e^{-6t}u(t)$ using the analysis equation. Show all your work.

(b) (6 points) Find $X_2(\omega)$ given $x_2(t) = e^{-6|t|}$. Use any method, but show your work.

(c) (8 points) Give an expression for and plot $X_3(\omega)$ given $x_3(t) = \frac{\sin(4t)\sin(t)}{\pi^2 t^2}$. Use any method, but show your work. Label key points on your plot.

(d) (8 points) Give an expression for and plot $X_4(\omega)$ given $x_4(t) = \frac{\sin(4t)\sin(t)}{\pi^2 t}$. Use any method, but show your work. Label key points on your plot.

a) $$X_1(\omega) = \int_{-\infty}^{\infty} x_1(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-6t} e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-t(6+j\omega)} dt = \frac{-1}{6+j\omega}\left[e^{-t(6+j\omega)}\right]_0^{\infty}$$

$$= \frac{-1}{6+j\omega}\left[e^{-\infty} e^{-\infty(j\omega)} - e^{0}\right] = \frac{1}{6+j\omega}$$

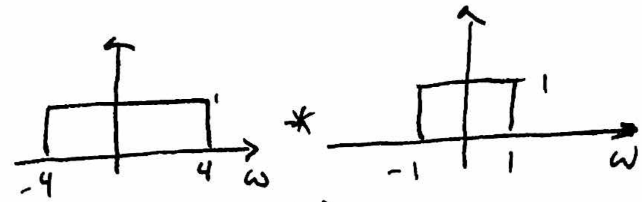
b) $x_2(t) = x_1(t) + x_1(-t)$

$$X_2(\omega) = X_1(\omega) + X_1(-\omega) = \frac{1}{6+j\omega} + \frac{1}{6-j\omega}$$

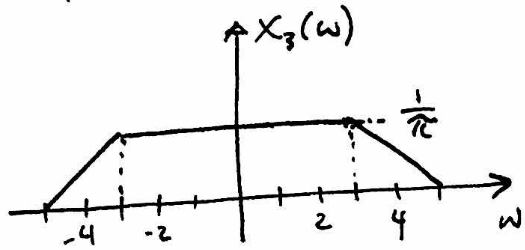
$$= \frac{6-j\omega+6+j\omega}{(6+j\omega)(6-j\omega)} = \frac{12}{36+\omega^2}$$

c) $x_3(t) = \frac{\sin 4t}{\pi t} \cdot \frac{\sin t}{\pi t}$

$X_3(\omega) = \frac{1}{2\pi} \cdot$

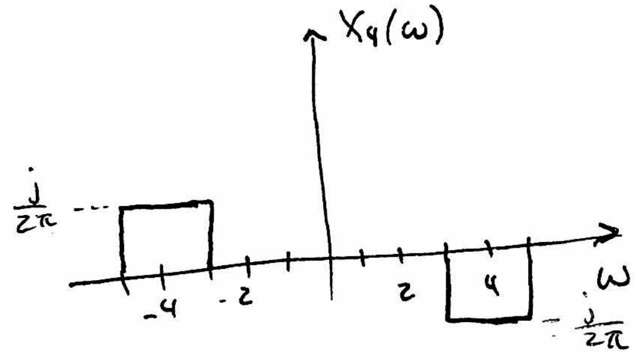


$=$



$$\hat{X}_3(\omega) = \begin{cases} \frac{1}{2\pi}\omega + \frac{5}{2\pi}, & -5 < \omega < -3 \\ -\frac{1}{2\pi}\omega + \frac{5}{2\pi}, & 3 < \omega < 5 \\ 1, & -3 < \omega < 3 \\ 0, & |\omega| > 5 \end{cases}$$

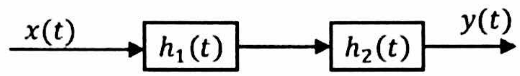
d) $x_4(t) = t\, x_3(t) \Rightarrow X_4(\omega) = j \frac{d}{d\omega} X_3(\omega)$



$$X_4(\omega) = \begin{cases} j\frac{1}{2\pi}, & -5 < \omega < -3 \\ -j\frac{1}{2\pi}, & 3 < \omega < 5 \\ 0, & \text{else} \end{cases}$$

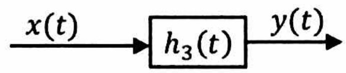
4. **CTFT and LTI Systems**

Given the system



where $h_1(t) = \frac{\sin(5t)}{\pi t}\cos(2t)$ and $h_2(t) = \frac{\sin(7t)}{\pi t}$.

(a) (6 points) Find an equivalent system with the impulse response $h_3(t)$.



(b) (6 points) Let $x(t) = \sum_{k=1}^{3} 2^k \cos(4kt)$. Find $X(\omega)$.

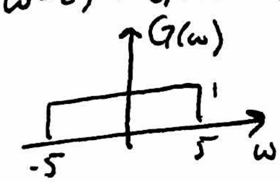
(c) (6 points) Find an expression for and plot $Y(\omega)$.

(d) (6 points) Find a simple expression for $y(t)$.

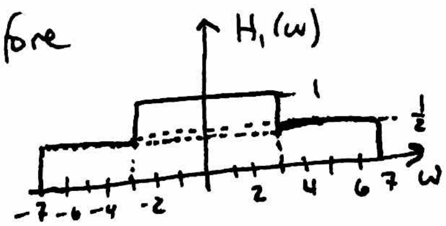
(e) (4 points) What restriction could you place on $x(t)$ so that $y(t) = x(t)$? In other words, what kind of signal will pass through the system unchanged.

a) $H_1(\omega) = \frac{1}{2}\left(G(\omega-2) + G(\omega+2)\right)$ by modulation,

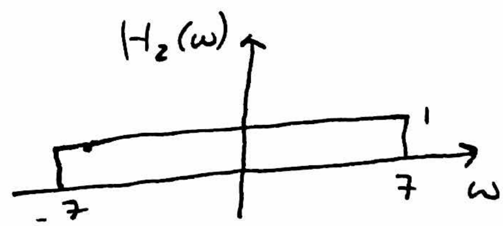
where



therefore



and



$H_3(\omega) = H_1(\omega) H_2(\omega) = H_1(\omega)$

$\Rightarrow$ $\boxed{h_3(t) = h_1(t) = \frac{\sin 5t}{\pi t}\cos 2t}$

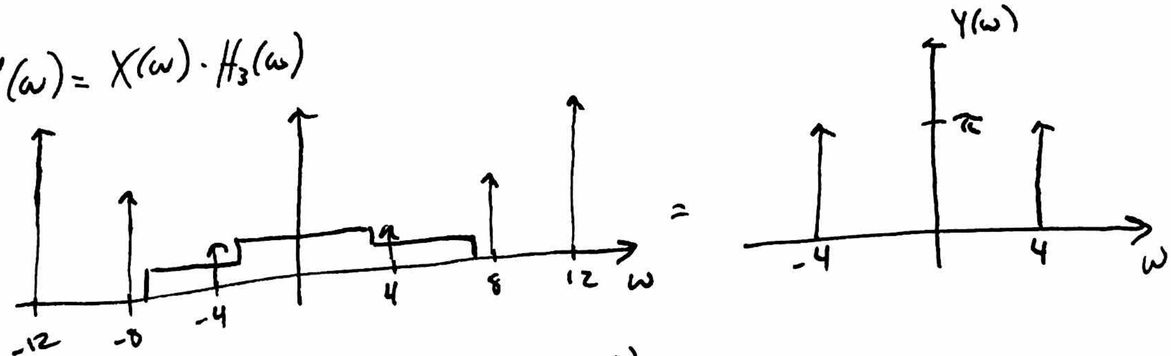
Equivalently, $h_3(t) = \frac{1}{2}\frac{\sin 7t}{\pi t} + \frac{1}{2}\frac{\sin 3t}{\pi t}$

b) $X(\omega) = \mathcal{F}\left\{\sum_{k=1}^{3} 2^k \cos(4kt)\right\} = \sum_{k=1}^{3} 2^k \mathcal{F}\{\cos(4kt)\}$

$$= \sum_{k=1}^{3} 2^k \pi\left(\delta(\omega-4k)+\delta(\omega+4k)\right)$$

$$= 2\pi\left(\delta(\omega-4)+\delta(\omega+4)\right) + 4\pi\left(\delta(\omega-8)+\delta(\omega+8)\right) + 8\pi\left(\delta(\omega-12)+\delta(\omega+12)\right)$$

c) $Y(\omega) = X(\omega)\cdot H_3(\omega)$



$$Y(\omega) = \pi\left(\delta(\omega-4)+\delta(\omega+4)\right)$$

d) From Table 4.2

$$y(t) = \cos(4t)$$

e) If $x(t)$ is bandlimited such that $X(\omega)=0$ for $|\omega| \geq 3$, then $y(t) = x(t)$.