

1. Prove or disprove: if f_n, g_n are uniformly convergent sequences on a set E then $f_n g_n$ is also.
2. Recall $C[0, 1]$ is a complete space with the sup norm, $\|f\| := \sup |f|$. In particular, $C[0, 1]$ is a metric space with $d(f, g) := \|f - g\|$. An easy way to remember Arzela-Ascoli's theorem is that it gives sufficient conditions for compactness of subsets of $C[0, 1]$. Prove this; namely,
 - (a) Show that if $A \subset C[0, 1]$ is closed, bounded (both in the sup norm), and equicontinuous, A is compact.
 - (b) Exhibit an A which is not compact and which satisfies any two of these three conditions. (hint, $A := \{f_n(x) = \frac{1}{nx + 1}\}$ is closed (every point is isolated!) and bounded but not compact (hence not equicontinuous!)).