(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No	
If $y(t) = x(2t)$, is the system causal?	✓		✗
If $y(t) = (t+2)x(t)$, is the system causal?		✓	✗
If $y(t) = x(-t^2)$, is the system causal?	✓		✗
If $y(t) = x(t) + t - 1$, is the system memoryless?	✓		
If $y(t) = x(t^2)$, is the system memoryless?		✓	
If $y(t) = x(t/3)$, is the system stable?	✓		
If $y(t) = tx(t/3)$, is the system stable?		✓	
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?		✓	
If $y(t) = \sin(x(t))$, is the system time invariant?	✓		
If $y(t) = u(t) * x(t)$, is the system LTI?	✓		
If $y(t) = (tu(t)) * x(t)$, is the system linear?	✓		

$x(t)\ \boxed{TD} \xrightarrow{y(t)=x(t-t_0)} \boxed{Sys}\ z(t) = \sin(y(t)) = \sin(x(t-t_0))$

$||$

$\boxed{Sys}\ y(t)=\sin(x(t)) \rightarrow \boxed{TD}\ z(t) = y(t-t_0) = \sin(x(t-t_0))$

15

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t-(\tau+2))\, d\tau$$

$u(\tau) = 1$ when $\tau > 0$

else 0

$$= \int_{0}^{\infty} e^{-\tau}\, u(t-(\tau+2))\, d\tau$$

$t-\tau-2 > 0 \Rightarrow u(t-\tau-2) = 1$

$t-2 > \tau$

else 0

$$= \begin{cases} \int_0^{t-2} e^{-\tau}\, d\tau & t-2 \geq 0 \\ & t \geq 2 \\ 0 & \text{else} \end{cases}$$

$$\int_0^{t-2} e^{-\tau}\, d\tau = -e^{-\tau}\Big|_0^{t-2} = -\left[e^{-t+2} - 1\right]$$

$$\boxed{y(t) = \begin{cases} -e^{-t+2} + 1 & t \geq 2 \\ 0 & \text{else} \end{cases}}$$

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(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| \frac{3e^{jt}}{1+j} \right|^2 dt$$

$$= \left|\frac{3}{1+j}\right|^2 \int_{-\infty}^{\infty} |e^{jt}|^2 = \left(\frac{3}{1+j}\right|^2 \int_{-\infty}^{\infty} 1^2$$

$$= \frac{9}{(1+j)^2} \; t \Big|_{-\infty}^{\infty}$$

$$= \frac{9}{(1+j)^2} \left(\infty - (-\infty)\right)$$

$$= \infty$$

$$\boxed{E_\infty = \infty}$$

$$T = \frac{2\pi}{1} = 2\pi$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{4\pi} \left|\frac{3}{1+j}\right|^2 \int_{-2\pi}^{2\pi} |e^{jt}|^2 \, dt$$

$$\int_{-2\pi}^{2\pi} 1 \, dt$$

$$t \Big|_{-2\pi}^{2\pi}$$

$$\frac{1}{4\pi} \left|\frac{3}{1+j}\right|^2 [4\pi]$$

4

$$\boxed{P_\infty = \frac{9}{(1+j)^2}}$$

4

(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$T = 4 \quad \omega_0 = \frac{2\pi}{4} \quad \cancel{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{4}t}}$

$$a_k = \frac{1}{4}\int_0^4 x(t) e^{-jk\left(\frac{2\pi}{4}\right)t}\,dt$$

$$a_k = \frac{1}{4}\int_0^2 \sin(\pi t) e^{-jk\frac{\pi}{2}t} + \cancel{\int_2^4 0\, e^{-jk\left(\frac{2\pi}{4}\right)t}}^{\,0}\,dt$$

$$a_k = \frac{1}{4}\int_0^2 \sin(\pi t)\, \cancel{j e^{kt}}\,dt$$

$a_k =$

$$a_0 = \frac{1}{4}\int_0^2 \sin(\pi t)\,dt$$

$$= \frac{1}{4}\left[-\frac{1}{\pi}\cos(\pi t)\Big|_0^2\right]$$

$$= \frac{-1}{4\pi}\left(\underset{1}{\cos 2\pi} - \underset{1}{\cos 0}\right)$$

$$\boxed{a_0 = 0} \qquad \boxed{a_k = \frac{j}{4}\int_0^2 \sin(\pi t) e^{kt}\,dt}$$

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5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$
\begin{array}{rcl}
\text{input} & & \text{output} \\
x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\
x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\
x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\
x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\
 & \vdots & \\
x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.}
\end{array}
$$

(10 pts) **a)** Can this system be time-invariant? Explain.

$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$ $\therefore$ $x[n] = u[n]$ $\quad y[n] = K \cdot u[n]$

~~so the~~ $y[n]$ is just multiplied by some constant everytime

$x[n] \rightarrow \boxed{TD} \xrightarrow{y[n] = x[n-n_0]} \boxed{sys} \rightarrow z[n] = y[kn] = x[kn-n_0] \neq$

$x[n] \rightarrow \boxed{sys} \xrightarrow{y[n] = x[kn]} \boxed{TD}\ z[n] = y[n-n_0] = x[k(n-n_0)]$ $\therefore$ the system <u>cannot</u> be Time invariant

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

$\cancel{\boxed{x[n] = u[n]}}$