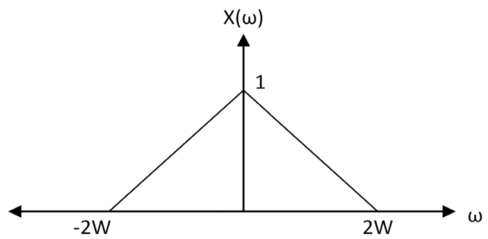
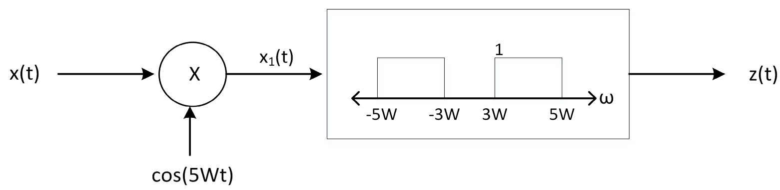
ECE 301: Homework 6

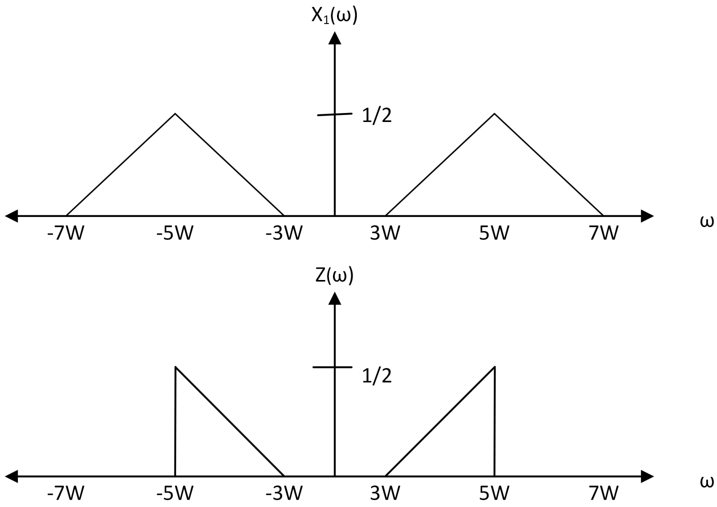
SOLUTIONS

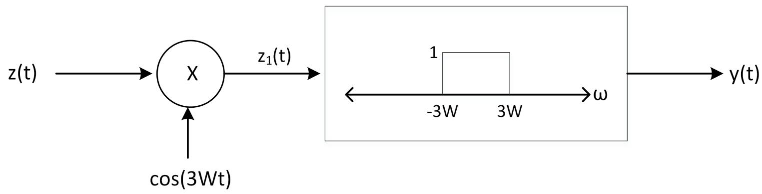
Problem 1



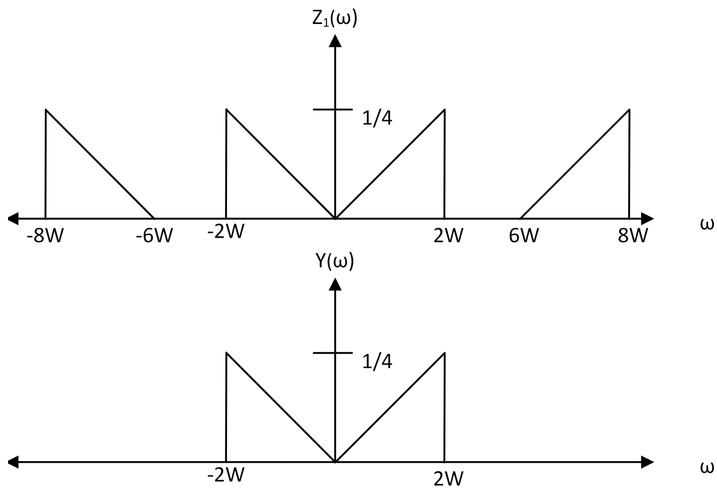


$$x_1(t) = x(t)cos(5Wt)$$
$$X_1(\omega) = \frac{1}{2}X(\omega - 5W) + \frac{1}{2}X(\omega + 5W)$$





$$Z_1(t) = Z(t)cos(3Wt)$$
$$Z_1(\omega) = \frac{1}{2}Z(\omega - 3W) + \frac{1}{2}Z(\omega + 3W)$$



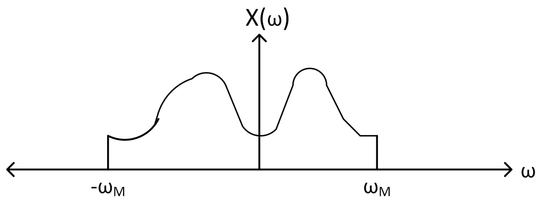
Problem 2

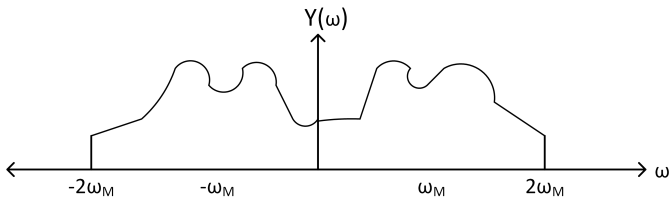
$X(j\omega) = 0$ when $|\omega| > \omega_M$

$$Y(j\omega) = \begin{cases} X(j(\omega - \omega_M)) & \text{if } \omega > 0 \\ X(j(\omega + \omega_M)) & \text{if } \omega < 0 \end{cases}$$

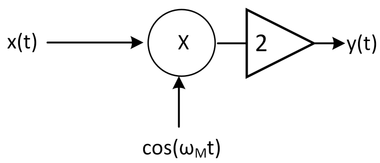
Therefore: $Y(j\omega) = X(\omega - \omega_M) + X(\omega + \omega_M)$ since $X(\omega)$ is band limited in ω_M

Part a:

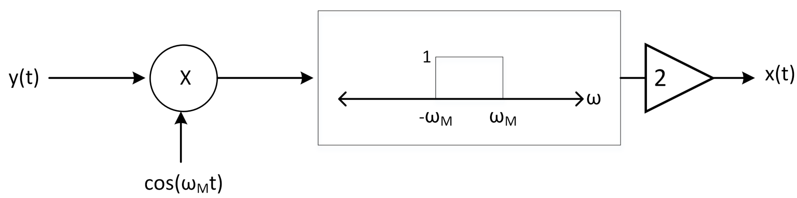




Part b:

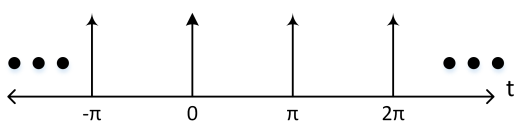


Part c:



Problem 3

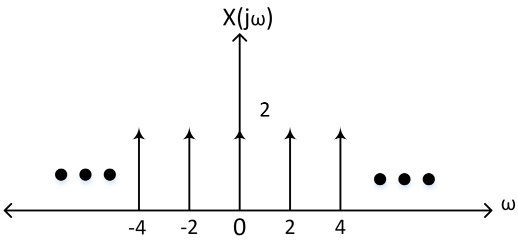
$y(t) = \sum_{k=-\infty}^{\infty} \delta(t - \pi k)$



x(t) is periodic with period π

Find the CTFS

$a_k = \frac{1}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \delta(t)e^{-jk2t}dt = \frac{1}{\pi}$



$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi}{T}k)$

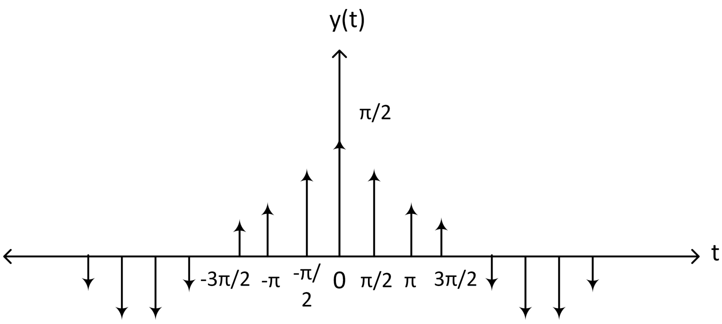
$$= \sum_{k=-\infty}^{\infty} 2\delta(\omega - 2k)$$

Problem 4

Part a:

$y(t) = \frac{\sin(\frac{t}{2})}{\pi t} \sum_{k=-\infty}^{\infty} \delta(t - \frac{\pi}{2}k)$

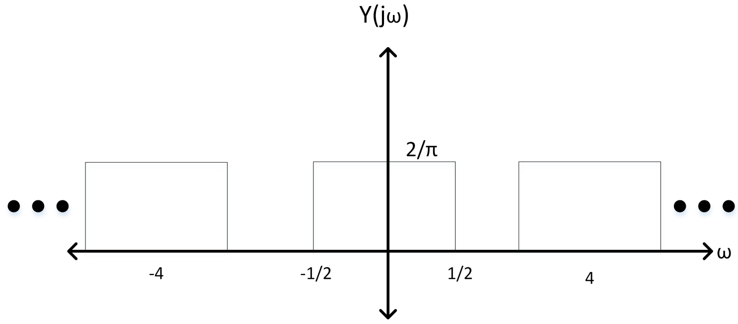
$$= \sum_{k=-\infty}^{\infty} \frac{\sin(\frac{\pi}{4}k)}{\frac{\pi^2}{2}k}\delta(t - \frac{\pi}{2}k)$$



Part b:

$Y(j\omega) = \frac{1}{2\pi}X(j\omega) * P(j\omega)$
$X(j\omega) = 1$ when $|\omega| < \frac{1}{2}$ and 0 otherwise.
$P(j\omega) = \frac{2\pi}{\frac{\pi}{2}} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi}{\frac{\pi}{2}}k) = 4\sum_{k=-\infty}^{\infty} \delta(\omega - 4k)$

$$Y(j\omega) = \frac{2}{\pi} \sum_{k=-\infty}^{\infty} X(j(\omega - 4k))$$



Problem 5

$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$ T=10^{-4}

Part a:

$X(j\omega) = 0$ for $|\omega| > 5000\pi$
Max frequency $= \frac{1}{2}(\frac{2\pi}{T}) = 10000\pi$
This signal can be perfectly reconstructed

Part b:

$X(j\omega) = 0$ for $|\omega| > 15000\pi$
This signal can NOT be perfectly reconstructed

Part f:

$X(j\omega) * X(j\omega) = 0$ for $|\omega| > 15000\pi$
$=> X(j\omega) = 0$ for $|\omega| > \frac{1}{2}(15000\pi) = 7500\pi$
This signal can be perfectly reconstructed

Part g:

$|X(j\omega)| = 0$ for $|\omega| > 5000\pi$
$|X(j\omega)| > 0$ for $|\omega| < -10000\pi$
This signal can NOT be perfectly reconstructed

Problem 6

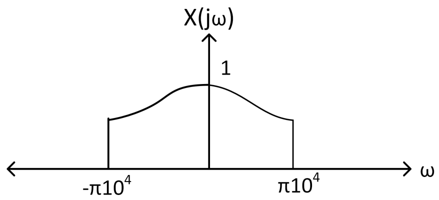
$y(t) = x_1(t) * x_2(t)$
$X_1(j\omega) = 0$ for $|\omega| > 1000\pi$
$X_2(j\omega) = 0$ for $|\omega| > 2000\pi$
$y_p(t) = \sum_{n=-\infty}^{\infty} y(nT)\delta(t - nT)$

$Y(j\omega) = 0$ for $|\omega| > 1000\pi$

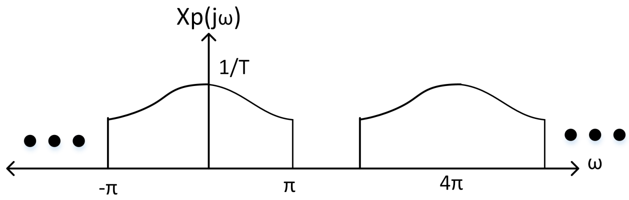
since $Y(j\omega) = X_1(j\omega)X_2(j\omega)$
$\omega_s = \frac{2\pi}{T} \geq 2(1000\pi)$

$$=> T \leq \frac{2\pi}{2000\pi} = 10^{-3}s$$

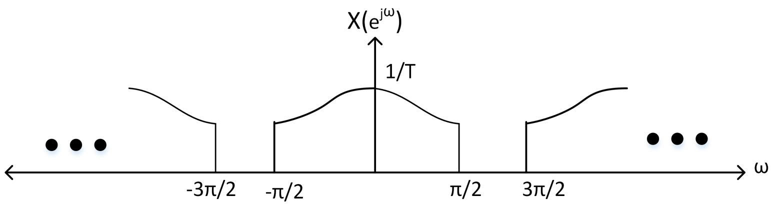
Problem 7



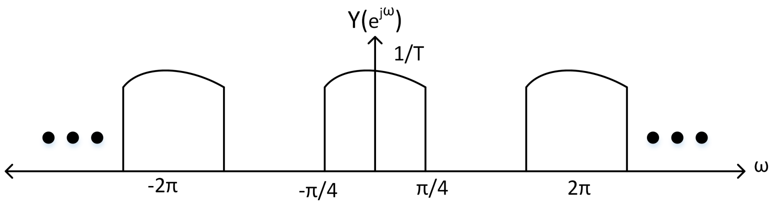
$$\frac{2\pi}{T} = 2\pi(20kHz) = 40\pi(1000) = 4\pi 10^4$$
$$P(j\omega) = \frac{2\pi}{T}\sum_n \delta(\omega - \frac{2\pi}{T}n)$$

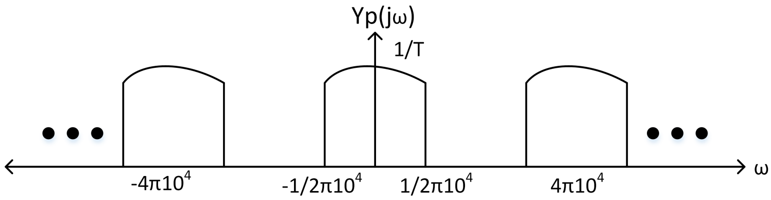


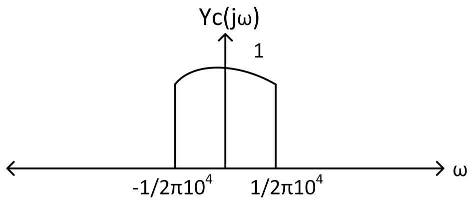
$$\frac{\pi}{T} = 2\pi x 10^4 => \pi \text{ in } X(e^{j\omega})$$
$$\pi x 10^4 => \frac{\pi}{2}$$



$$\pi => \frac{\pi}{T} \text{ in } Y_p(j\omega)$$
$$\frac{\pi}{4} => \frac{\pi}{4T} \text{ in } Y_p(j\omega)$$
$$\frac{\pi}{4T} = \frac{\pi}{4}(20000) = 50x10^3 = 0.50x10^4$$

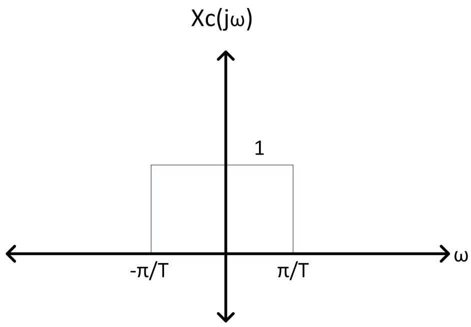






Problem 8

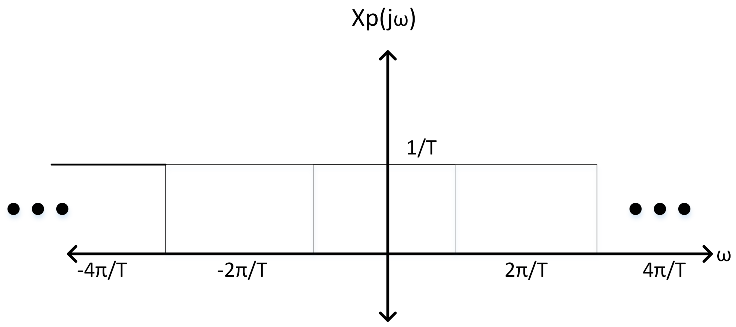
Let:

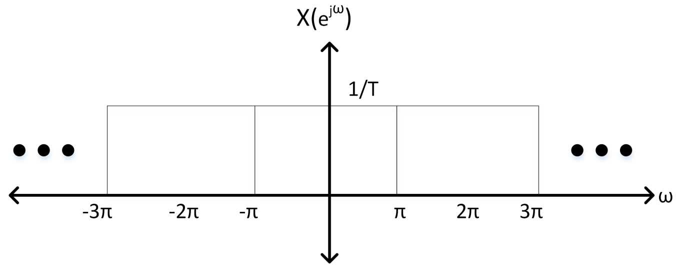


Then $H_c(j\omega) = \frac{Y_c(j\omega)}{X_c(j\omega)} = Y(j\omega)$

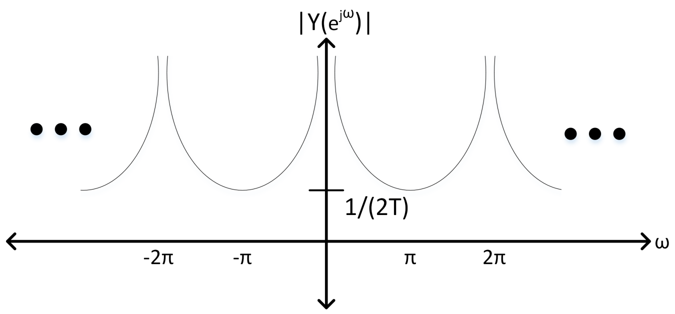
$$Y(e^{j\omega}) = \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) + X(j\omega)$$

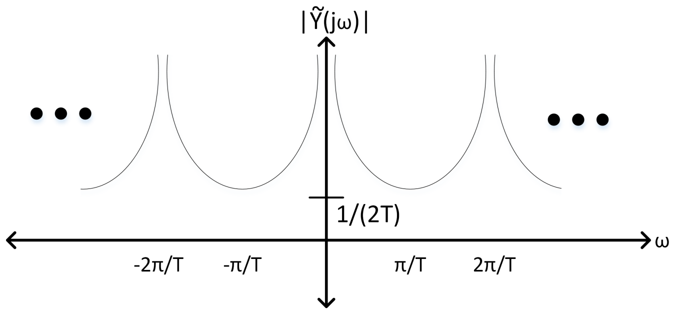
$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$



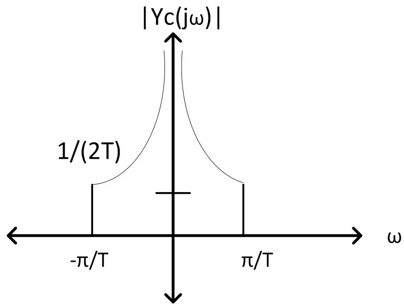


$$
\begin{aligned}
|H(e^j\omega)| &= \frac{1}{\sqrt{(1-\frac{1}{2}\cos(\omega))^2+\sin^2(\omega)}} \\
&= \frac{1}{\sqrt{2-2\cos(\omega)}}
\end{aligned}
$$





$$Y(e^{j\omega}) = \frac{1}{T} H(e^{j\omega})$$



$$Y_c(j\omega) = \frac{1}{T}\frac{1}{1-\frac{1}{2}e^{-j\omega T}} \text{ for } |\omega| < \frac{\pi}{T}$$
$$Y_c(j\omega) = 0 \text{ for } |\omega| > \frac{\pi}{T}$$

$$H_c(j\omega) = \begin{cases} \frac{1}{T}\frac{1}{1-\frac{1}{2}e^{-j\omega T}} & \text{if } |\omega| < \frac{\pi}{T} \\ 0 & \text{if } |\omega| > \frac{\pi}{T} \end{cases}$$