1) $x[n] = \left(\frac{1}{a_i}\right)^{|n|}$

$$= \begin{cases} \frac{1}{a_i} & n \geq 0 \\ \left(\frac{1}{a_i}\right)^{-n} & n < 0 \end{cases}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-i\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{a_i}\right)^{|n|} e^{-i\omega n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{1}{a_i}\right)^{-n} e^{-i\omega n} + \sum_{n=0}^{\infty} \left(\frac{1}{a_i}\right)^{n} e^{-i\omega n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{e^{i\omega}}{a_i}\right)^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{a_i e^{i\omega}}\right)^{n}$$

let $K = -n$

$$= \sum_{k=1}^{\infty} \left(\frac{e^{i\omega}}{a_i}\right)^{K} + \sum_{n=0}^{\infty} \left(\frac{1}{a_i e^{i\omega}}\right)^{n}$$

$$\boxed{= \frac{1}{1 - \frac{e^{i\omega}}{a_i}} - 1 + \frac{1}{1 - \frac{1}{a_i e^{i\omega}}}}$$

2) $y(t) = x(t) * h(t) = \mathcal{F}^{-1}\left(X(\omega) H(\omega)\right)$ pr

so need $X(\omega) = \mathcal{F}\left(e^{-|t|}\right)$

$= \mathcal{F}\left(e^{-t}u(t) + e^{t}u(-t)\right)$

$= \mathcal{F}\left(e^{-t}u(t)\right) + \mathcal{F}\left(e^{-t}u(t)\right)\Big|_{\omega \to -\omega}$ by 13

$= \frac{1}{1+j\omega} + \frac{1}{1-j\omega}$, by 7.

so $y(t) = \mathcal{F}^{-1}\left(\left(\frac{1}{1+j\omega} + \frac{1}{1-j\omega}\right)\frac{1}{j\omega+2}\right)$

$= \mathcal{F}^{-1}\left(\frac{1}{1+j\omega}\frac{1}{j\omega+2} + \frac{1}{1-j\omega}\frac{1}{j\omega+2}\right)$

$= \mathcal{F}^{-1}\left(\frac{1}{1+j\omega} + \frac{-1}{j\omega+2} + \frac{1/3}{1-j\omega} + \frac{1/3}{j\omega+2}\right)$

$$\boxed{y(t) = e^{-t}u(t) - e^{-2t}u(t) + \frac{1}{3}e^{t}u(-t) + \frac{1}{3}e^{-2t}u(t)}$$

3. True

The F.T. of a DT signal is periodic wi[th]
period 2π because it is a linear combin[ation]
of functions which are periodic with peri[od]

$$X(\omega+2\pi) = \sum_{n=-\infty}^{\infty} x[n] e^{-j(\omega+2\pi)n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} e^{-j2\pi n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= X(\omega)$$

4) $H(j\omega) = \frac{Y(\omega)}{X(\omega)}$

$$\frac{j\omega+4}{(j\omega)^2+5j\omega+6} = \frac{Y(\omega)}{X(\omega)}$$

$$(j\omega+4)X(\omega) = ((j\omega)^2+5j\omega+6)Y(\omega)$$

taking F.T. and using 16

$$x'(t)+4x(t) = y''(t)+5y'(t)+6y(t)$$

5) by 10.

$$|y(\omega)| = |X(\omega)|$$

because

$$y(\omega) = e^{-j\omega 6} X(\omega)$$

$$|X(\omega)| = |-2e^{(j-1)\omega} u(\omega+1)|$$

$$= 2|e^{j\omega} e^{-\omega}| u(\omega+1)$$

$$= 2e^{-\omega} u(\omega+1)$$