HW1 Solution

1.48 Given z_0 as a complex number with polar coordinates (r_0, θ_0)

(a) We can express $z_0 = r_0 e^{j\theta_0}$

$$= r_0 \cos\theta_0 + j r_0 \sin\theta_0$$

$$= x_0 + j y_0$$

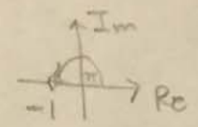
Similarly, for $z_1 = r_0 e^{-j\theta_0}$

$$= r_0 \cos\theta_0 - j r_0 \sin\theta_0$$

$$= x_0 - j y_0$$

(c) $z_3 = r_0 e^{j(\theta_0 + \pi)}$

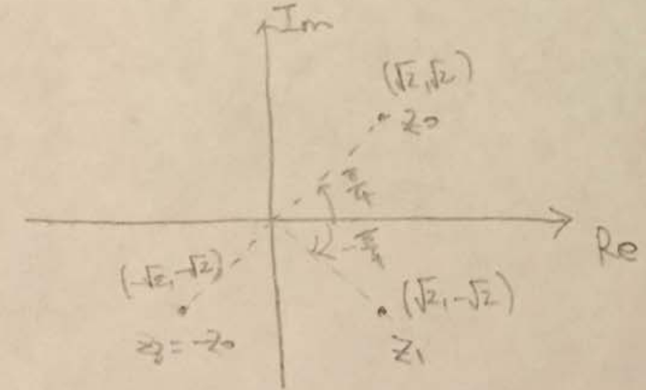
$$= r_0 e^{j\theta_0} \cdot \underbrace{e^{j\pi}}_{=-1}$$

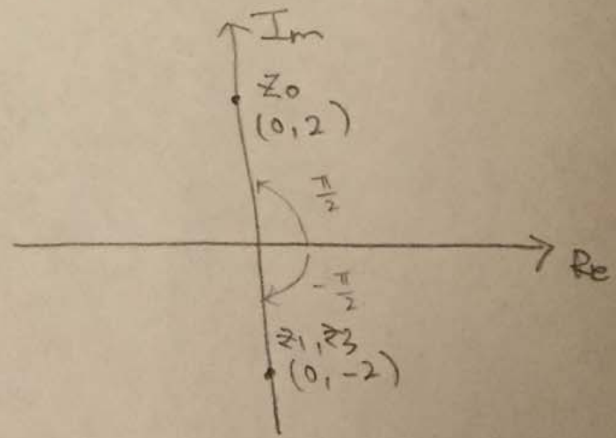


$$= -r_0 e^{j\theta_0}$$

$$= -x_0 - j y_0$$

$$= -z_0$$





$\langle r_0 = 2, \ \theta_0 = \frac{\pi}{4} \rangle$ $z_0 = 2e^{j\frac{\pi}{4}}$

(a) $z_1 = 2e^{-j\frac{\pi}{4}}$

(c) $z_3 = -2e^{j\frac{\pi}{4}}$

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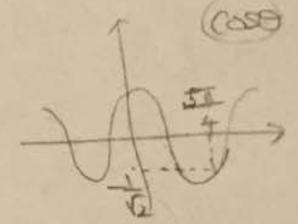
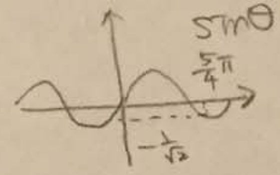
c) $-5-5j$

$r = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$



$$\left.\begin{aligned} r\cos\theta &= -5 \\ \cos\theta &= \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} \\ jr\sin\theta &= -5j \\ \sin\theta &= -\frac{1}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \theta = \frac{5}{4}\pi$$



$\therefore$ In polar form, $5\sqrt{2}e^{j\frac{5}{4}\pi}$

f) $(1+j)^5$

$(1+j)^2 = 1+2j+\underbrace{j^2}_{-1} = 2j$

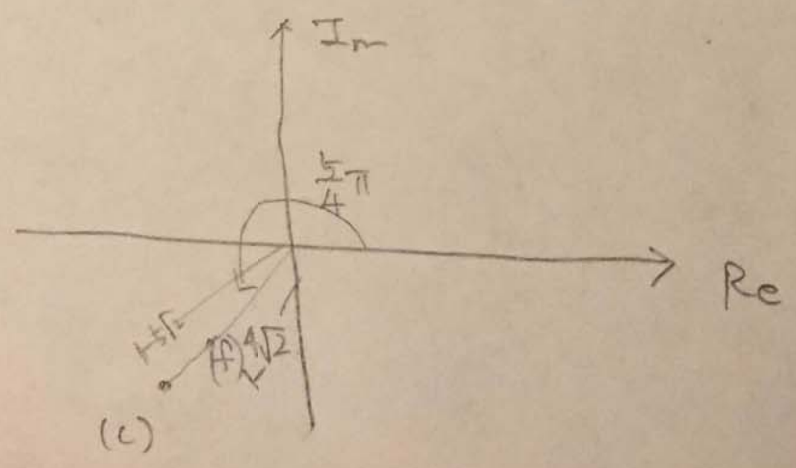
$(1+j)^4 = (2j)^2 = 4j^2 = -4$

$\therefore (1+j)^5 = -4-4j$

$r = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$$\left.\begin{aligned} r\cos\theta = -4 &\rightarrow \cos\theta = \frac{-4}{4\sqrt{2}} = -\frac{1}{\sqrt{2}} \\ jr\sin\theta = -4j &\rightarrow \sin\theta = -\frac{1}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \theta = \frac{5}{4}\pi$$

$\therefore$ In polar form, $4\sqrt{2}e^{j\frac{5}{4}\pi}$



(h) $$\frac{2 - j\left(\frac{6}{\sqrt{3}}\right)}{2 + j\left(\frac{6}{\sqrt{3}}\right)} = \frac{2 - j(2\sqrt{3})}{2 + j(2\sqrt{3})}$$

$$= \frac{(2 - j(2\sqrt{3}))(2 - j2\sqrt{3})}{(2 + j2\sqrt{3})(2 - j2\sqrt{3})} = \frac{4 - j8\sqrt{3} - 12}{16}$$

$$= \frac{-8 - j8\sqrt{3}}{16}$$

$$= -\frac{1}{2} - \frac{j\sqrt{3}}{2}$$

$$r = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1.$$

$$\left.\begin{aligned} r\cos\theta &= -\frac{1}{2} \rightarrow \cos\theta = -\frac{1}{2} \\ jr\sin\theta &= -\frac{j\sqrt{3}}{2} \rightarrow \sin\theta = -\frac{\sqrt{3}}{2} \end{aligned}\right\} \Rightarrow \theta = -\frac{2}{3}\pi$$

$\therefore$ In polar coordinates, $e^{-j\frac{2}{3}\pi}$

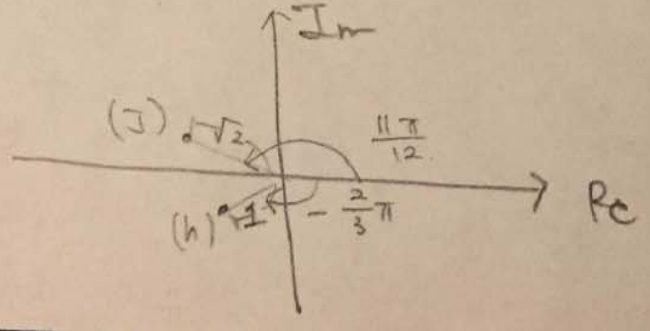
(j) $$j(1+j)\,e^{j\frac{\pi}{6}} = (j + j^2)\,e^{j\frac{\pi}{6}}$$

$$= (j-1)\,e^{j\frac{\pi}{6}} = je^{j\frac{\pi}{6}} - e^{j\frac{\pi}{6}}$$

$$r = \sqrt{1+1} = \sqrt{2}$$

$$\left.\begin{aligned} -e^{j\frac{\pi}{6}} &= -\cos\left(\frac{\pi}{6}\right) - j\sin\left(\frac{\pi}{6}\right) \\ &= -\frac{\sqrt{3}}{2} - j\frac{1}{2} \\ je^{j\frac{\pi}{6}} &= j\cos\left(\frac{\pi}{6}\right) - j^2\sin\left(\frac{\pi}{6}\right) \end{aligned}\right\} \Rightarrow \theta = \frac{11\pi}{12}$$

$\therefore$ In polar coordinates, $\sqrt{2}e^{j\frac{11\pi}{12}}$



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a) $\sum_{n=0}^{9} e^{\frac{j\pi n}{2}} = \sum_{n=0}^{9} e^{\left(\frac{j\pi}{2}\right)^n}$

$$= \frac{1-e^{\frac{j\pi 10}{2}}}{1-e^{\frac{j\pi}{2}}} = \frac{2}{1-j} = \frac{2(1+j)}{(1-j)(1+j)} = \frac{2+2j}{1-j^2} = 1+j$$

c) $\sum_{n=-2}^{7} e^{\frac{j\pi n}{2}} = e^{-\frac{j2\pi}{2}} \sum_{n=0}^{9} e^{\left(\frac{j\pi}{2}\right)^n}$

$$= (-1)\sum_{n=0}^{9} e^{\left(\frac{j\pi}{2}\right)^n} = -1(1+j)$$

$$= -1-j$$

d) $\sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^n e^{\frac{j\pi n}{2}} = \left(\frac{1}{2}\right)^2 e^{j\frac{2\pi}{2}} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{\frac{j\pi n}{2}}$

$$= \frac{1}{4}(-1) \frac{1}{1-\left(\frac{1}{2}\right)e^{\frac{j\pi}{2}}} \rightarrow \frac{1}{1-\frac{j}{2}} = \frac{1+\frac{j}{2}}{\left(1-\frac{j}{2}\right)\left(1+\frac{j}{2}\right)}$$

$$= \frac{4+2j}{5}$$

$$= -\frac{1}{4}\left(\frac{4}{5} + j\frac{2}{5}\right)$$

1.56

(a) $\int_0^4 e^{\frac{J\pi t}{2}}\,dt = \left[\dfrac{e^{\frac{\pi t}{2}}}{\frac{J\pi}{2}}\right]_0^4$

$$= \frac{e^{\frac{4\pi}{2}}}{\frac{J\pi}{2}} - \frac{1}{\frac{J\pi}{2}} = \frac{\overset{=1}{e^{2\pi}} - 1}{\frac{J\pi}{2}} = 0$$

(c) $\int_2^8 e^{\frac{J\pi t}{2}}\,dt = \left[\dfrac{e^{\frac{\pi t}{2}}}{\frac{J\pi}{2}}\right]_2^8$

$$= \frac{e^{4\pi}}{\frac{J\pi}{2}} - \frac{e^{\pi}}{\frac{J\pi}{2}}$$

$$= \frac{1-(-1)}{\frac{J\pi}{2}} = \frac{2}{\frac{J\pi}{2}} = \frac{4}{J\pi}$$

$$= \frac{4}{J\pi}\frac{(J)}{(J)} = \frac{4J}{J^2\pi} = \frac{4J}{-\pi}$$

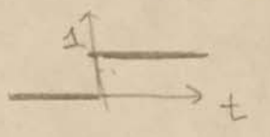
$$= -\frac{4J}{\pi}$$

1.3

(a) $x_1(t) = e^{-2t} u(t)$

$$E_\infty = \int_0^\infty e^{-4t}\,dt = \left[-\frac{1}{4}e^{-4t}\right]_0^\infty = 0 - \left(-\frac{1}{4}\right) = \frac{1}{4}$$

due to $u(t) \rightarrow$ step function



$P_\infty = 0$ since $E_\infty < \infty$

(b) $x_2(t) = e^{j(2t + \frac{\pi}{4})}$

$|x_2(t)| = 1$

$$E_\infty = \int_{-\infty}^{\infty} 1^2\,dt = [t]_{-\infty}^{\infty} = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_2(t)|^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} dt = \lim_{T\to\infty} 1 = 1$$

(d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

$|x_1[n]|^2 = \left(\frac{1}{4}\right)^n u[n]$

$E_\infty = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{4}{3}$, $P_\infty = 0$ since $E_\infty < \infty$

(f) $f_3[n] = \cos\left(\frac{\pi}{4}n\right)$

$$E_\infty = \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{\pi}{4}n\right) = \infty$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4}n\right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} \left(\frac{1+\cos\left(\frac{\pi}{2}n\right)}{2}\right) = \frac{1}{2}$$