EE 301 Midterm Exam #1

September 29, Fall 2005

Name: ____________________________________

Instructions:

- Follow all instructions carefully!
- This is a 60 minute exam containing **four** problems totaling 100 points.
- You **may not** use a calculator.
- You **may not** use any notes, books or other references.
- You may only keep pencils, pens and erasers at your desk during the exam.

Good Luck.

Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1 - |t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFS

$$\begin{aligned} a_k &= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j2\pi kt/T}dt \\ x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T} \end{aligned}$$

- CTFT

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\ x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \end{aligned}$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-\omega)$$

$$x(t-t_0) \overset{CTFT}{\Leftrightarrow} X(\omega)e^{-j\omega t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|}X(\omega/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} 2\pi x(-\omega)$$

$$x(t)e^{j\omega_0 t} \overset{CTFT}{\Leftrightarrow} X(\omega-\omega_0)$$

$$x(t)y(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{2\pi}X(\omega) * Y(\omega)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(\omega)Y(\omega)$$

$$\frac{dx(t)}{dt} \overset{CTFT}{\Leftrightarrow} j\omega X(\omega)$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

If $x(t) \overset{CTFS}{\Leftrightarrow} a_k$ then

$$x(t) \overset{CTFT}{\Leftrightarrow} \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - 2\pi k/T)$$

- CTFT pairs

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(\omega/(2\pi))$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(\omega/(2\pi))$$

For $a > 0$

$$\frac{1}{(n-1)!}t^{n-1}e^{-at}u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{(j\omega + a)^n}$$

$$\sum_{k-\infty}^{\infty} \delta(t-kT) \overset{CTFT}{\Leftrightarrow} \frac{1}{T}\sum_{k-\infty}^{\infty} 2\pi\delta(\omega-2\pi k/T)$$

- DFT

$$\begin{aligned} X_k &= \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \\ x(n) &= \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N} \end{aligned}$$

- DTFT

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega \end{aligned}$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1 - ae^{-j\omega}}$$

- Sampling and Reconstruction

$$\begin{aligned} y(n) &= x(nT) \\ Y(\omega) &= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{T}\right) \\ s(t) &= \sum_{k=-\infty}^{\infty} y(k)\delta(t-kT) \\ S(\omega) &= Y(\omega T) \end{aligned}$$

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Problem 1.(20pt) *Formal Logic*

Define the two sets: $A = \{\text{set of Purdue students}\}$ and $B = \{\text{set of football games}\}$, and the logical property: Pxy is true if student x has attended game y.

a) Give a formal logical expression which is equivalent to the sentence: "All Purdue students have attended a football game."

b) Give the logical negation of the expression from part a) above.

c) What must be done to prove that the statement "All Purdue students have attended a football game." is false?

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Problem 2.(25pt) *Sinusoidal Inputs to LTI Systems*

Consider the system $y(t) = T[x(t)]$ with input $x(t) = e^{j\omega_1 t} + e^{j\omega_2 t}$ where $\omega_1 \neq \omega_2 \neq 0$.

a) If the system is LTI, then what is the most general form of the output $y(t)$?

b) If $y(t) = e^{j\omega_1 t}e^{j\omega_2 t}$, then is it possible that the system is LTI? Prove your answer.

c) If $y(t) = e^{j\omega_1 t}$, then is it possible that the system is LTI? Prove your answer.

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Problem 3.(25pt) *LTI Systems*

Consider the LTI system $y(t) = T[x(t)]$ where

$$\frac{dy(t)}{dt} = ay(t) + x(t)$$

where the system is assumed to be initially at rest (i.e. $\lim_{t \to -\infty} x(t) = \lim_{t \to -\infty} y(t) = 0$).

a) Find the impulse response of the system.

b) Give the condition for BIBO stability of the system.

c) For what values of a is the system BIBO stable? Be precise.

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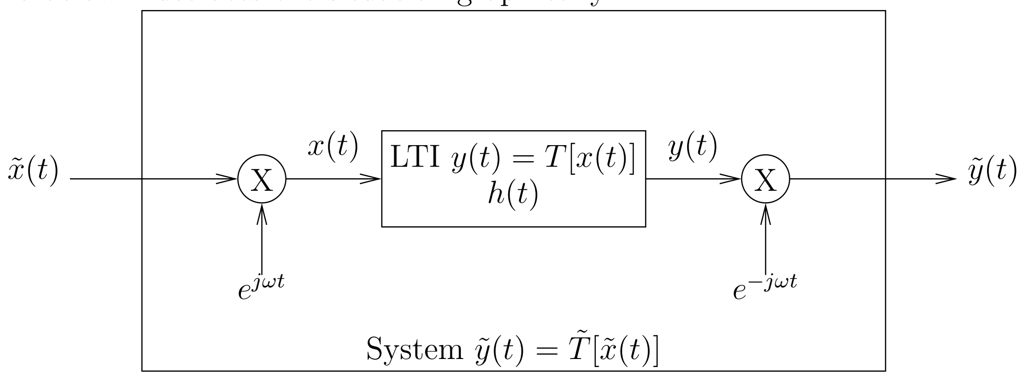
Problem 4.(30pt) *LTI Systems*

Consider a continuous time LTI system $y(t) = T[x(t)]$ with input $x(t)$, output $y(t)$, and impulse response $h(t)$. Furthermore, consider the system $\tilde{y}(t) = \tilde{T}[\tilde{x}(t)]$ where $x(t) = e^{j\omega t}\tilde{x}(t)$ and $\tilde{y}(t) = e^{-j\omega t}y(t)$ and ω is a constant.

The figure below illustrates the situation graphically.



a) Write an expression for $y(t)$ in terms of the functions $x(t)$ and $h(t)$.

b) Write an expression for $\tilde{y}(t)$ in terms of the functions $\tilde{x}(t)$ and $h(t)$.

c) Either prove that the system $\tilde{y}(t) = \tilde{T}[\tilde{x}(t)]$ is LTI, or prove it is not LTI.

d) Find the impulse response of the system $\tilde{T}[\tilde{x}(t)]$.

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