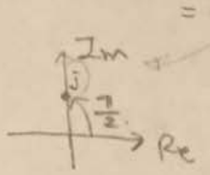
HW 2 Solution

1.9

(a) $x_1(t) = j e^{j10t}$

$= e^{j\frac{\pi}{2}} e^{j10t} = e^{j(10t + \frac{\pi}{2})}$



$$T_0 = \frac{2\pi}{|w_0|} = \frac{2\pi}{10} = \frac{\pi}{5}$$

$x_1(t)$ is periodic with fundamental period $= \frac{\pi}{5}$

(b) $x_2(t) = e^{(-1+j)t}$

$= \underbrace{e^{-t}}_{\text{decaying exponential}} \underbrace{e^{jt}}_{\text{complex}} \Rightarrow x_2(t)$ is not periodic

(c) $x_3[n] = e^{j7\pi n}$ periodic

$= e^{j\pi n}$

$$N = m\left(\frac{2\pi}{w_0}\right) = m\left(\frac{2\pi}{\pi}\right)$$

by choosing $m = 1$, fundamental period $= 2$

(d) $x_4[n] = e^{\frac{j3\pi(n+\frac{1}{2})}{5}}$

$$N = m\left(\frac{2\pi}{3\pi/5}\right) = m\frac{10}{3}$$ by choosing $m = 3$,

fundamental period $= 10$. periodic

(e) $x_5[n] = 3e^{\frac{j3}{5(n+\frac{1}{2})}}$

$N = m\left(\frac{2\pi}{\frac{3}{5}}\right) \Rightarrow$ No integer m can make N to be an integer.

$\therefore x_5[n]$ is not periodic

1.10

$x(t) = 2\cos(10t+1) - \sin(4t-1)$

$\underbrace{2\cos(10t+1)}$: Period $= \frac{2\pi}{|w_0|} = \frac{2\pi}{10} = \frac{\pi}{5}$

$\underbrace{\sin(4t-1)}$: period $= \frac{2\pi}{|w_0|} = \frac{2\pi}{4} = \frac{\pi}{2}$

↳ $x(t)$ is periodic with the least common multiple of two periods, $\frac{\pi}{2}$ and $\frac{\pi}{5}$

$$\frac{10}{10}\pi = \pi$$

1.11

$x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2\pi n}{5}}$

↳ $e^{j0n} = e^{j2\pi n}$

$N = m\left(\frac{2\pi}{2\pi}\right) = 1$, for $m=1$

$N = m\left(\frac{2\pi}{4\pi/7}\right) = 7$, for $m=2$

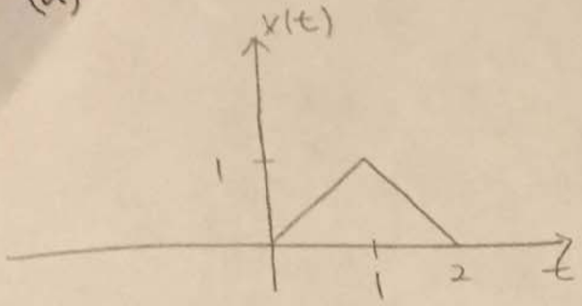
$N = m\left(\frac{2\pi}{2\pi/5}\right) = 5$, for $m=1$

$\Rightarrow x[n]$ is periodic with the least common multiple of three periods 1, 5 and 7

which is 35

1.23

(a)

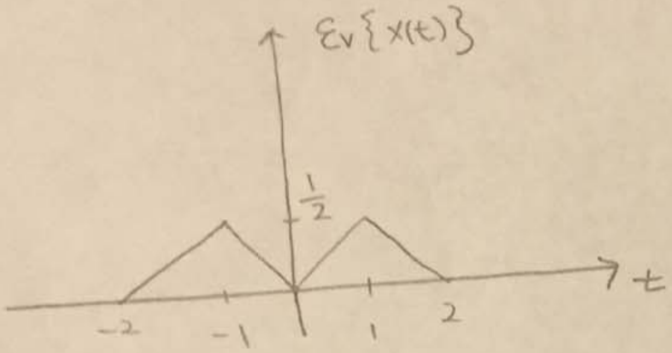


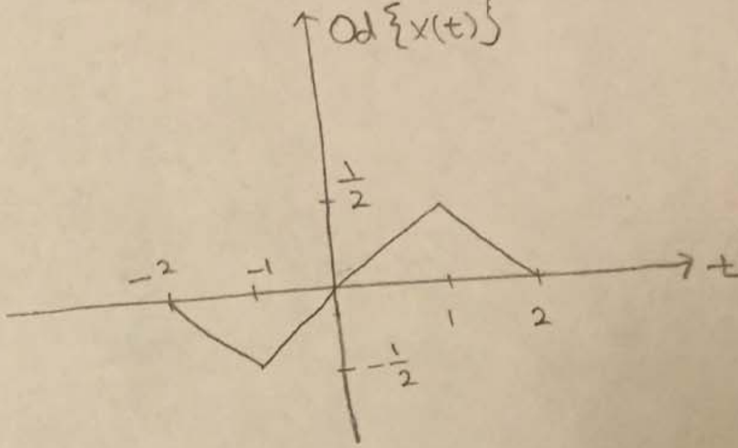
even

$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$Od\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right]$$

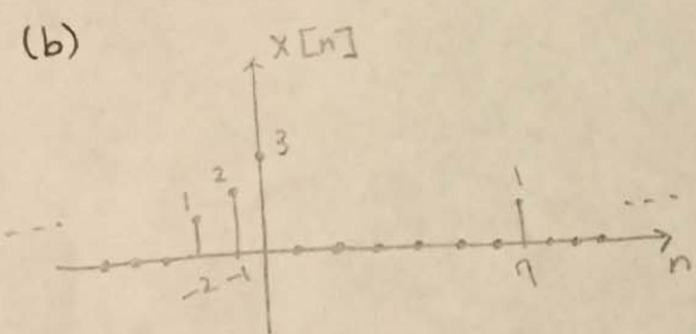
odd





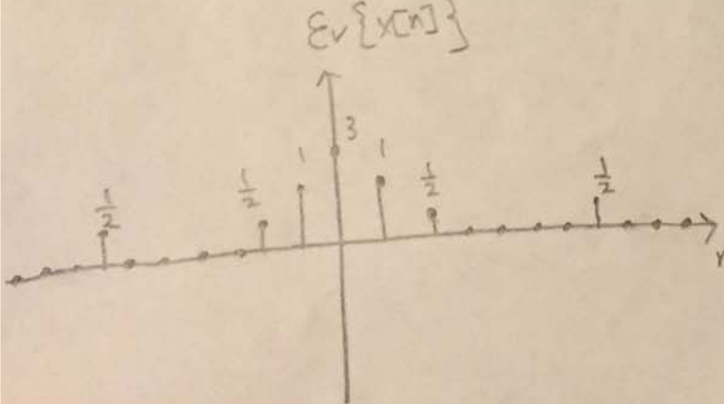
1.24

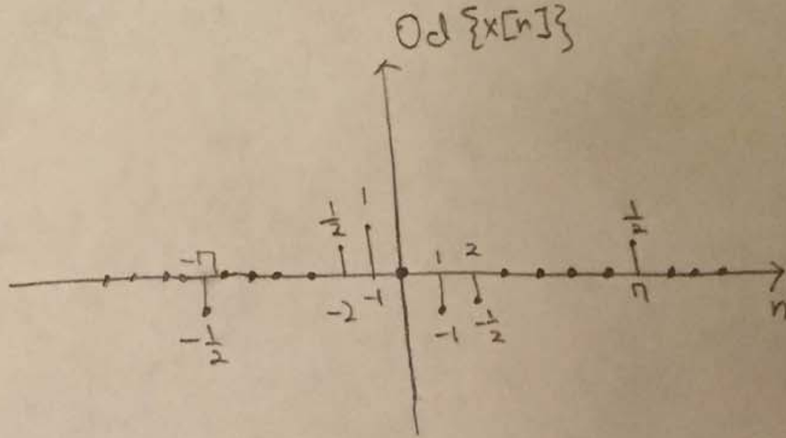
(b)



$$\mathcal{E}v\{x[n]\} = \frac{1}{2}\left[x[n] + x[-n]\right]$$

$$Od\{x[n]\} = \frac{1}{2}\left[x[n] - x[-n]\right]$$





1.36

$x(t) = e^{jW_0 t}$

$T_0 = \frac{2\pi}{W_0}$

$x[n] = x(nT) = e^{jW_0 nT}$

(a) If $x[n]$ is periodic, $e^{jW_0(n+N)T} = e^{jW_0 nT}$ where $W_0 = \frac{2\pi}{T_0}$

$\frac{2\pi}{T_0} NT = 2\pi k$

$\frac{T}{T_0} = \frac{k}{N}$ = rational number

(b) fundamental frequency $= \frac{2\pi}{N_0} = \frac{2\pi}{q} \cdot \overset{\text{greatest common divisor}}{gcd}(p, q)$

$= \frac{2\pi}{p} \frac{p}{q} gcd(p, q)$

$= \frac{2\pi}{p} \frac{T}{T_0} gcd(p, q)$

$= \underline{\frac{W_0 T}{p} gcd(p, q)}$ $\left[\because \frac{2\pi}{T_0} = W_0\right]$

$\therefore$ fundamental period $= N_0 = \frac{q}{gcd(p, q)}$

(c) $\frac{p}{gcd(p, q)}$ periods are needed to obtain the samples that form a single period of $x[n]$.

1.17.

$y(t) = x(\sin(t))$

(a) System is not causal because the output $y(t)$ may depend on future input values.

ex) $y(-\pi) = x(\sin(-\pi))$

$= x(0).$

$\Rightarrow$ the output $y(-\pi)$ depends on future $x(0)$.

(b) Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$

$x_2(t) \rightarrow y_2(t) = x_2(\sin(t))$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$.

Then, $x_3(t) = a x_1(t) + b x_2(t)$, where a and b are arbitrary scalars,

If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t) = x_3(\sin(t))$

$= a x_1(\sin(t)) + b x_2(\sin(t))$

$= a y_1(t) + b y_2(t)$

$\therefore$ The system is linear.

1.27

(b) $y(t) = [\cos(3t)]x(t)$

$x_1(t) \rightarrow y_1(t) = [\cos(3t)]\, x_1(t)$

$x_2(t) \rightarrow y_2(t) = [\cos(3t)]\, x_2(t)$

↳ $x_3(t) = ax_1(t) + bx_2(t)$, where a and b are arbitrary scalars.

$$y_3(t) = [\cos(3t)]\, x_3(t)$$
$$= ax_1(t)[\cos(3t)] + b x_2(t)[\cos(3t)]$$
$$= ay_1(t) + by_2(t)$$

∴ <u>Linear.</u>

<u>Causal</u> since the output only depends on present and past time input.

<u>not Time invariant</u> because the output vary with time.

$y(t)$ is in the form of $Rx(t)$, where $R = \cos(3t)$

∴ <u>memoryless</u>

<u>stable</u> since $\cos(3t)$ does not diverge, the output $y(t)$ is stable.

1.27

(b) $y(t) = [\cos(3t)]\, x(t)$

Let $y_1(t) = [\cos(3t)]\, x_1(t)$

$y_2(t) = [\cos(3t)]\, x_2(t)$

$x_3(t) = a x_1(t) + b x_2(t)$, where a and b are arbitrary scalars.

$y_3(t) = [\cos(3t)]\, x_3(t)$

$= a x_1(t)[\cos(3t)] + b x_2(t)[\cos(3t)]$

$= a y_1(t) + b y_2(t)$ $\therefore$ Linear

Causal output only depends on present and past time input.

not time invariant shifted output $y(t-t_0) = [\cos(3(t-t_0))]\, x(t-t_0)$

shifted input $x_d(t) = x(t-t_0)$

$y_d(t) = [\cos(3t)]\, x_d(t)$

$= [\cos(3t)]\, x(t-t_0)$

$\neq y(t-t_0)$

memoryless

$y(t)$ is in $Bx(t)$ form.

Stable

$|x(t)| < B$

$-B < x(t) < B$

$|y(t)| < 2B$

$-B < y(t) < 2B$

(d) $y(t) = \begin{cases} 0 & , t < 0 \\ x(t) + x(t-2) & , t \geq 0 \end{cases}$

We can express $y(t) = \{x(t) + x(t-2)\} u(t)$

Let

$y_1(t) = \{x_1(t) + x_1(t-2)\} u(t)$

$y_2(t) = \{x_2(t) + x_2(t-2)\} u(t)$

Let $x_3(t) = ax_1(t) + bx_2(t)$

$y_3(t) = \{x_3(t) + x_3(t-2)\} u(t) = a\{x_1(t) + x_1(t-2)\} u(t) + b\{x_2(t) + x_2(t-2)\} u(t)$

$y_3(t) = ay_1(t) + by_2(t)$

$\therefore$ Linear

Causal output depends on present and past time input.

not time invariant

shifted output $y(t-t_0) = \left(x(t-t_0) + x(t-t_0-2)\right) u(t-t_0)$

shifted input $x_d(t) = x(t-t_0)$

$y_d(t) = \left(x_d(t) + x_d(t-2)\right) u(t)$

$= \left(x(t-t_0) + x(t-t_0-2)\right) u(t)$

$\neq y(t-t_0)$

not memoryless.

not in $Rx(t)$ form.

$y(t)$ depends on current time and the time "2" earlier.

$\hookrightarrow$ It has memory.

Stable

$y(t) = 0$ for $t < 0$

$|x(t)| < B$

for $t \leq 0$

$|y(t)| < 2B$

1.27

(e) $y(t) = \begin{cases} 0 & , x(t) < 0 \\ x(t) + x(t-2) & , x(t) \geq 0 \end{cases}$

$x_s(t) = ax_1(t) + bx_2(t)$

$$y_s(t) = S\{x_s(t)\} = (x_s(t) + x_s(t-2))\,u(x_s(t))$$

$$= (ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2))\,u(ax_1(t) + bx_2(t))$$

$$= a(x_1(t) + x_1(t-2))\,u(ax_1(t) + bx_2(t)) + b(x_2(t) + x_2(t-2))\,u(ax_1(t) + bx_2(t))$$

$$\neq aS\{x_1(t)\} + bS\{x_2(t)\}$$

$$= a(x_1(t) + x_1(t-2))\,u(x_1(t)) + b(x_2(t) + x_2(t-2))\,u(x_2(t))$$

$\therefore$ nonlinear

Causal: output only depends on present and past input value.

Time invariant: shifted output $y(t-t_0) = (x(t-t_0) + x(t-t_0-2))\,u(x(t-t_0))$

shifted input $x_d(t) = x(t-t_0)$

$$y_d(t) = (x_d(t) + x_d(t-2))\,u(x_d(t))$$

$$= (x(t-t_0) + x(t-2-t_0))\,u(x(t-t_0))$$

$$= y(t-t_0)$$

not memoryless: $y(t)$ depends on $x(t-2)$

Stable

$$|x(t)| < B$$

$$-B < x(t) < B$$

$$|y(t)| < 2B$$

$$-B < y(t) < 2B$$

(f) $y(t) = x\left(\frac{t}{3}\right)$

Let

$y_1(t) = x_1\left(\frac{t}{3}\right)$

$y_2(t) = x_2\left(\frac{t}{3}\right)$

Let

$x_3(t) = ax_1(t) + bx_2(t)$ $\quad$ $x_4(t) = ax_1(t) + bx_2(t)$

$y_3(t) = x_3\left(\frac{t}{3}\right)$ $\quad$ $y_4(t) = x_3$

$= ax_1\left(\frac{t}{3}\right) + bx_2\left(\frac{t}{3}\right)$

$= ay_1(t) + by_2(t)$ $\therefore$ Linear

not causal $\quad$ ex) $y(-3) = x(-1)$

output depends on future input.

not time invariant

shifted output $\quad y(t-t_0) = x\left(\frac{t-t_0}{3}\right)$

shifted input $\quad x_d(t) = x(t-t_0)$

$y_d(t) = x_d(t)$

$= x\left(\frac{t}{3} - t_0\right)$

$\neq y(t-t_0)$

not memoryless $\quad$ (not in $Rx(t)$ form).

Stable

$|x(t)| < B$

$-B < x(t) < B$

$|y(t)| < 2B$

$-B < y(t) < 2B$

1.28

(a) $y[n] = x[-n]$

Let $y_1[n] = x_1[-n]$

$y_2[n] = x_2[-n]$

Let $x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = x_3[-n]$

$= ax_1[-n] + bx_2[-n]$

$= ay_1[n] + by_2[n]$ $\therefore$ Linear

not causal ex) $y[-1] = x[1]$

not time invariant

shifted output $y[n-n_0] = x[-(n-n_0)]$

shifted input $x_d[n] = x[n-n_0]$

$y_d[n] = x_d[-n]$

$= x[-n-n_0]$

$\neq y[n-n_0]$

not memoryless depends on $x[-n]$

stable

$|x[n]| < B$

$-B < x[n] < B$

$|y[n]| < 2B$

$-B < y[n] < 2B$

(b) $y[n] = x[n-2] - 2x[n-8]$

Let $y_1[n] = x_1[n-2] - 2x_1[n-8]$

$y_2[n] = x_2[n-2] - 2x_2[n-8]$

Let $x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = x_3[n-2] - 2x_3[n-8]$

$= ax_1[n-2] + bx_2[n-2] - 2ax_1[n-8] - 2bx_2[n-8]$ $\therefore$ Linear

Causal $= a[x_1[n-2] - 2x_1[n-8]] + b[x_2[n-2] - 2x_2[n-8]]$

$= ay_1[n] + by_2[n]$

Time invariant

shifted output $y[n-n_0] = x[n-n_0-2] - 2x[n-n_0-8]$

shifted input $x_d[n] = x[n-n_0]$

$y_d[n] = x_d[n-2] - 2x_d[n-8]$

$= x[n-n_0-2] - 2x[n-n_0-8$

$= y[n-n_0]$

not memoryless

$y[n]$ depends on $x[n-2]$ and $2x[n-8]$.

Stable

$|x[n]| < B$

$-B < x[n] < B$

$|y[n]| < 2B$

$-B < y[n] < 2B$

(f) $y[n] = \begin{cases} x[n] & , \quad n \geq 1 \\ 0 & , \quad n = 0 \\ x[n] & , \quad n \leq -1 \end{cases}$

$$y[n] = x[n]u[-n+1] + x[n]u[n-1]$$

Let $y_1[n] = x_1[n]u[-n+1] + x_1[n]u[n-1]$

$y_2[n] = x_2[n]u[-n+1] + x_2[n]u[n-1]$

Let $x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = x_3[n]u[-n+1] + x_3[n]u[n-1]$

$= [ax_1[n] + bx_2[n]]u[n+1] + [ax_1[n] + bx_2[n]]u[n-1]$

$= ax_1[n]u[-n+1] + ax_1[n]u[n-1] + bx_2[n]u[-n+1] + bx_2[n]u[n-1]$

causal

not time invariant

shifted output $\quad y[n-n_0] = x[n-n_0]u[-(n-n_0)+1] + x[n-n_0]u[n-n_0-1]$

shifted input $\quad x_d[n] = x[n-n_0]$

$y_d[n] = x_d[n]u[-n+1] + x_d[n]u[n-1]$

$= x[n-n_0]u[-n+1] + x[n-n_0]u[n-1]$

$\neq y[n-n_0]$

memoryless (in $kx[n]$ form)

Stable

$|x[n]| < B$

$-B < x[n] < B$

$|y[n]| < 2B$

$-B < y[n] < 2B$

(9) $y[n] = x[4n+1]$

Let $y_1[n] = x_1[4n+1]$

$y_2[n] = x_2[4n+1]$

Let $x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = x_3[4n+1]$

$= ax_1[4n+1] + bx_2[4n+1]$

$= ay_1[n] + by_2[n]$ $\therefore$ Linear.

not causal ex) $y[0] = x[1]$

not time invariant

shifted output $y[n-n_0] = x[4(n-n_0)+1]$

shifted input $x_d[n] = x[n-n_0]$

$y_d[n] = x_d[4n+1]$

$= x[4n+1-n_0]$

$\neq y[n-n_0]$

not memoryless (not in $kx[n]$ form)

Stable.

$x[n] \longrightarrow \boxed{\begin{array}{c} z[n] \\ x[4n] \end{array}} \longrightarrow \boxed{\begin{array}{c} w[n] \\ = x[n+1] \end{array}} \longrightarrow y[n]$