HW 2 Solution

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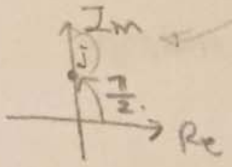
) $x_1(t) = j e^{j10t}$

$= e^{j\frac{\pi}{2}} e^{j10t} \quad = e^{j(10t + \frac{\pi}{2})}$



$$T_0 = \frac{2\pi}{|w_0|} = \frac{2\pi}{10} = \frac{\pi}{5}$$

$x_1(t)$ is periodic with fundamental period $= \frac{\pi}{5}$

$x_2(t) = e^{(-1+j)t}$

$= \underbrace{e^{-t}}_{} \underbrace{e^{jt}}_{\text{complex}}$

decaying exponential $\Rightarrow$ $x_2(t)$ is not periodic

) $x_3[n] = e^{j7\pi n}$ periodic

$= e^{j\pi n}$

$$N = m\left(\frac{2\pi}{w_0}\right) = m\left(\frac{2\pi}{\pi}\right)$$

by choosing $m = 1$, fundamental period $= 2$

) $x_4[n] = e^{\frac{j3\pi(n+\frac{1}{2})}{5}}$

$$N = m\left(\frac{2\pi}{3\pi/5}\right) = m\,\frac{10}{3}$$ by choosing $m=3$,

fundamental period $= 10$. periodic

) $x_5[n] = 3e^{\frac{j3}{5}(n+\frac{1}{2})}$

$$N = m\left(\frac{2\pi}{\frac{3}{5}}\right) \Rightarrow$$ No integer m can make N to be an integer.

$\therefore$ $x_5[n]$ is not periodic

1.10

$x(t) = \underbrace{2\cos(10t+1)} - \underbrace{\sin(4t-1)}$

$\rightarrow$ period $= \frac{2\pi}{|w_0|} = \frac{2\pi}{4} = \frac{\pi}{2}$

Period $= \frac{2\pi}{|w_0|} = \frac{2\pi}{10} = \frac{\pi}{5}$

$\rightarrow$ $x(t)$ is periodic with the least common multiple of two periods, $\frac{\pi}{2}$ and $\frac{\pi}{5}$

$$\frac{10}{10}\pi = \pi$$

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$x[n] = \underbrace{1} + \underbrace{e^{j\frac{4\pi n}{7}}} - \underbrace{e^{j\frac{2\pi n}{5}}}$

$\rightarrow$ $e^{j0n} = e^{j2\pi n}$

$N = m\left(\frac{2\pi}{2\pi}\right) = 1$, for $m=1$

$\rightarrow$ $N = m\left(\frac{2\pi}{4\pi/7}\right) = 7$, for $m=2$

$\rightarrow$ $N = m\left(\frac{2\pi}{2\pi/5}\right) = 5$, for $m=1$

$\Rightarrow$ $x[n]$ is periodic with the least common multiple of three periods 1, 5 and 7

which is 35

3

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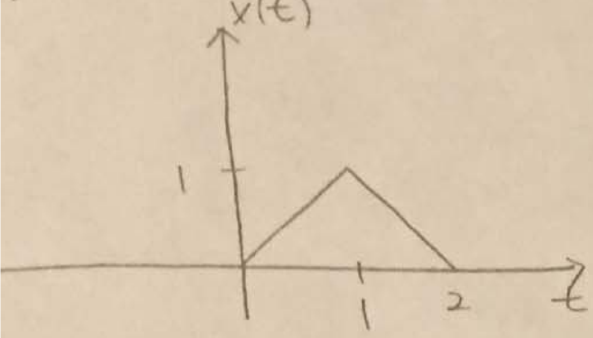
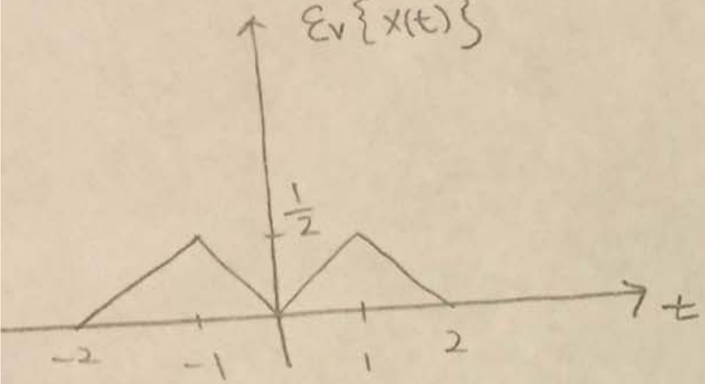


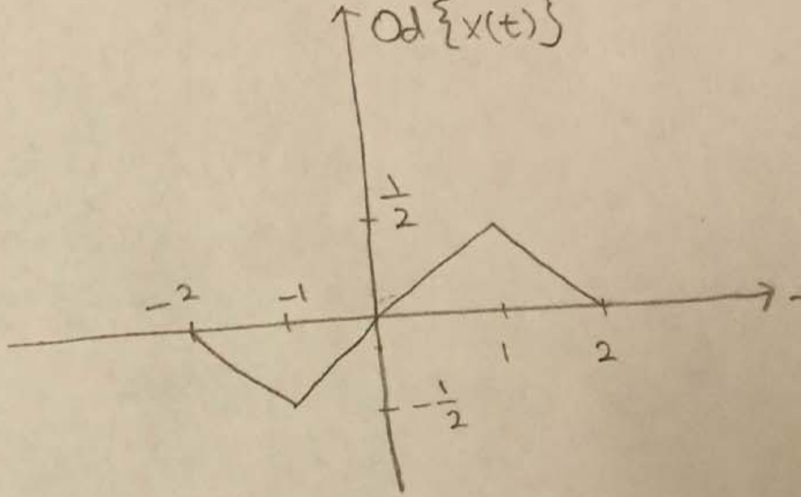
even

$\mathcal{E}v\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$

$\mathcal{O}d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$

odd





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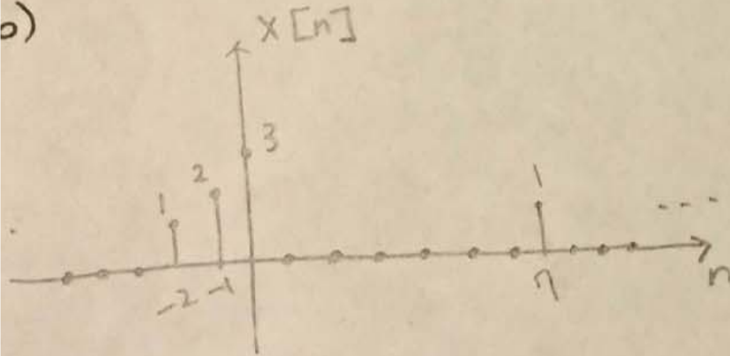
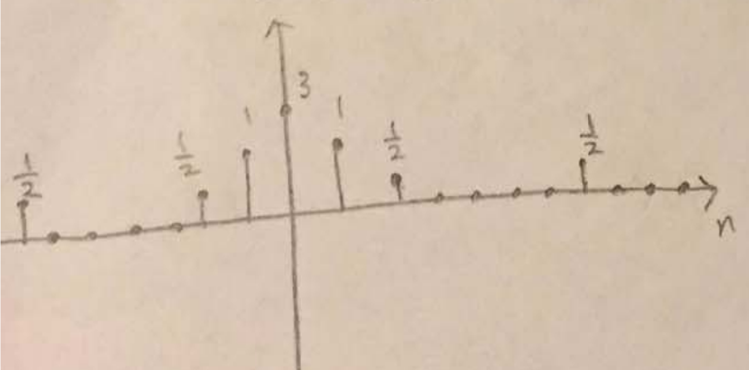
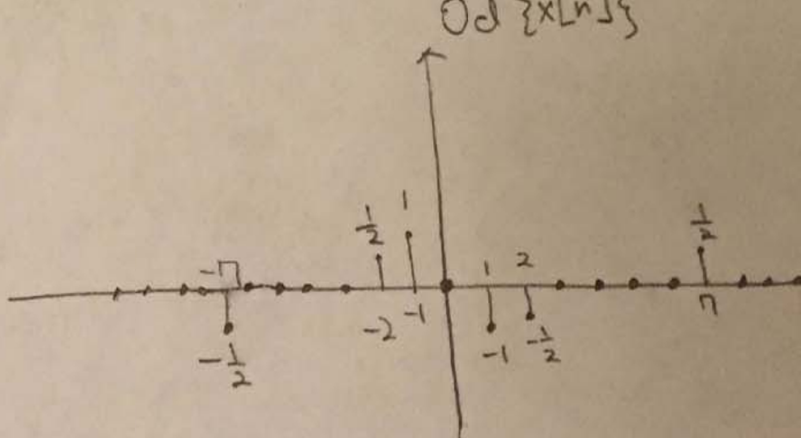


$\mathcal{E}v\{x[n]\} = \frac{1}{2}[x[n] + x[-n]]$

$\mathcal{O}d\{x[n]\} = \frac{1}{2}[x[n] - x[-n]]$





6

$x(t) = e^{jW_0 t}$

$T_0 = \frac{2\pi}{W_0}$

$x[n] = x(nT) = e^{jW_0 nT}$

) If $x[n]$ is periodic, $e^{jW_0(n+N)T} = e^{jW_0 nT}$ where $W_0 = \frac{2\pi}{T_0}$

$\frac{2\pi}{T_0} NT = 2\pi k$

$\frac{T}{T_0} = \frac{k}{N}$ = rational number

greatest common divisor

fundamental frequency $= \frac{2\pi}{N_0} = \frac{2\pi}{q} \cdot \gcd(P, q)$

$= \frac{2\pi}{P} \frac{P}{q} \gcd(P, q)$

$= \frac{2\pi}{P} \frac{T}{T_0} \gcd(P, q)$

$= \frac{W_0 T}{P} \gcd(P, q)$ $\left[\because \frac{2\pi}{T_0} = W_0\right]$

$\therefore$ fundamental period $= N_0 = \frac{q}{\gcd(P, q)}$

$\frac{P}{\gcd(P, q)}$ periods are needed to obtain the samples

that form a single period of $x[n]$.

17

$y(t) = x(\sin(t))$

(a) System is not causal because the output $y(t)$ may depend on future input values.

ex) $y(-\pi) = x(\sin(-\pi))$

$= x(0)$.

$\Rightarrow$ the output $y(-\pi)$ depends on future $x(0)$.

)

Consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

$x_1(t) \rightarrow y_1(t) = x_1(\sin(t))$

$x_2(t) \rightarrow y_2(t) = x_2(\sin(t))$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$.

Then, $x_3(t) = ax_1(t) + bx_2(t)$, where a and b are arbitrary scalars,

If $x_3(t)$ is the input to the given system, then the corresponding output $y_3(t) = x_3(\sin(t))$

$= ax_1(\sin(t)) + bx_2(\sin(t))$

$= ay_1(t) + by_2(t)$

$\therefore$ The system is linear.

2.17

b) $y(t) = [\cos(3t)]x(t)$

$x_1(t) \rightarrow y_1(t) = [\cos(3t)]x_1(t)$

$x_2(t) \rightarrow y_2(t) = [\cos(3t)]x_2(t)$

$\hookrightarrow x_3(t) = ax_1(t) + bx_2(t)$, where a and b are arbitrary scalars.

$y_3(t) = [\cos(3t)]x_3(t)$

$= ax_1(t)[\cos(3t)] + bx_2(t)[\cos(3t)]$

$= ay_1(t) + by_2(t)$

$\therefore$ Linear.

Causal since the output only depends on present and past time input.

not Time invariant because the output vary with time.

$y(t)$ is in the form of $Rx(t)$, where $R = \cos(3t)$

$\therefore$ memoryless

Stable since $\cos(3t)$ does not diverge, the output $y(t)$ is stable

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) $y(t) = \begin{cases} 0 & , t < 0 \\ x(t) + x(t-2) & , t \geq 0 \end{cases}$

$y_1(t) = x_1(t) + x_1(t-2)$

$y_2(t) = x_2(t) + x_2(t-2)$

Let

$$x_3(t) + x_3(t-2) = a\{x_1(t) + x_1(t-2)\} + b\{x_2(t) + x_2(t-2)\}$$

$$y_3(t) = a\{x_1(t) + x_1(t-2)\} + b\{x_2(t) + x_2(t-2)\}$$
$$= a y_1(t) + b y_2(t)$$

$\therefore$ Linear

causal . output only depends on presend and past time in

not time invariant (output vary with time)

not memoryless. (not in $Rx(t)$ form)

stable (does not diverse)

) $y(t) = \begin{cases} 0 & , x(t) < 0 \\ x(t) + x(t-2) & , x(t) \geq 0 \end{cases}$

given that $x(t) < 0$ makes $y(t) = 0$, this system is nonlinear.

Causal. Output only depends on present and past time in

Time invariant Output does not vary with time.
It only depends on input value.

not memoryless Not in $Rx(t)$ form.

stable

$y(t) = x\left(\frac{t}{3}\right)$

Let $y_1(t) = x_1\left(\frac{t}{3}\right)$
$y_2(t) = x_2\left(\frac{t}{3}\right)$

Let $x_3\left(\frac{t}{3}\right) = a x_1\left(\frac{t}{3}\right) + b x_2\left(\frac{t}{3}\right)$

$y_3(t) = a x_1\left(\frac{t}{3}\right) + b x_2\left(\frac{t}{3}\right)$
$= a y_1(t) + b y_2(t)$ $\therefore$ Linear

not causal ex) $y(-3) = x(-1) \Longrightarrow$ output depends on future input.

not time invariant (output vary with time)

not memoryless (not in $Rx(t)$ form)

stable

$y[n] = x[-n]$

Let $y_1[n] = x_1[-n]$

$y_2[n] = x_2[-n]$

Let

$x_3[-n] = ax_1[-n] + bx_2[-n]$, where a and b are arbitrary s

$y_3[n] = ax_1[-n] + bx_2[-n]$

$= ay_1[n] + by_2[n]$

$\therefore$ Linear

not causal ex) $y[-1] = x[1]$

not time invariant

not memoryless

stable

$y[n] = x[n-2] - 2x[n-8]$

$y_1[n] = x_1[n-2] - 2x_1[n-8]$

$y_2[n] = x_2[n-2] - 2x_2[n-8]$

$x_3[n-2] - 2x_3[n-8] = a\{x_1[n-2] - 2x_1[n-8]\} + b\{x_2[n-2] - 2x_2[n-8]\}$

$= y_3[n] = ay_1[n] + by_2[n]$

$\therefore$ Linear

causal

Time invariant

not memoryless

stable

$$Y[n] = \begin{cases} x[n] &, n \geq 1 \\ 0 &, n = 0 \\ x[n] &, n \leq -1 \end{cases}$$

Let $y_1[n] = x_1[n]$
$y_2[n] = x_2[n]$

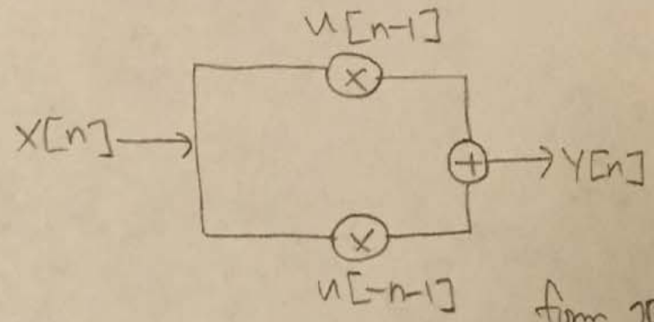
Let $x_3[n] = a x_1[n] + b x_2[n]$
$y_3[n] = a y_1[n] + b y_2[n]$ $\therefore$ Linear

Causal

not time invariant

memoryless (in $Rx[n]$ form) with $R=1$

stable



form 3(

$Y[n] = x[4n+1]$

Let $y_1[n] = x_1[4n+1]$
$y_2[n] = x_2[4n+1]$

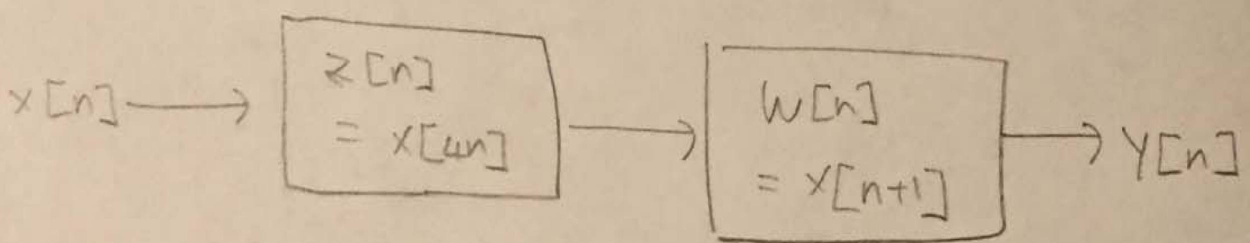
Let $x_3[4n+1] = a x_1[4n+1] + b x_2[4n+1]$
$y_3[n] = a y_1[n] + b y_2[n]$ $\therefore$ Linear

not causal ex) $Y[0] = x[1]$ output depends on future input

Time invariant

not memoryless (not in $Rx[n]$ form)

Stable



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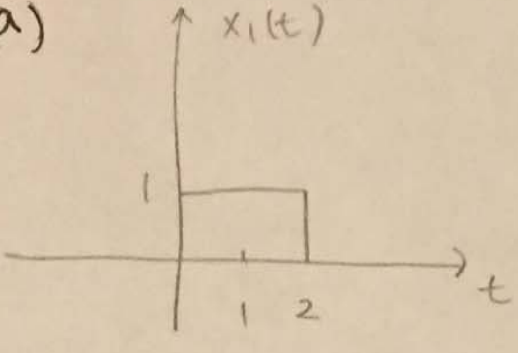
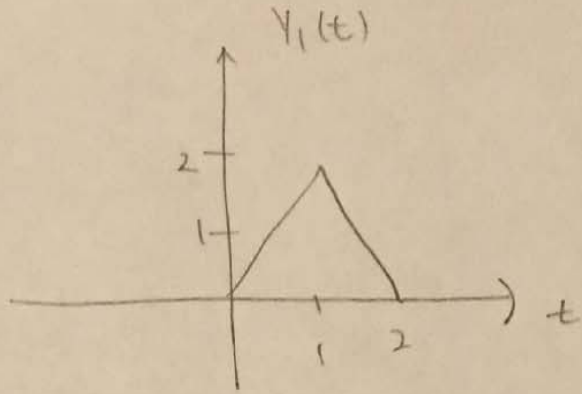
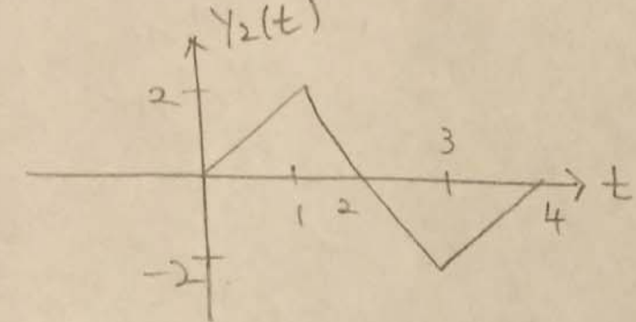
(a)





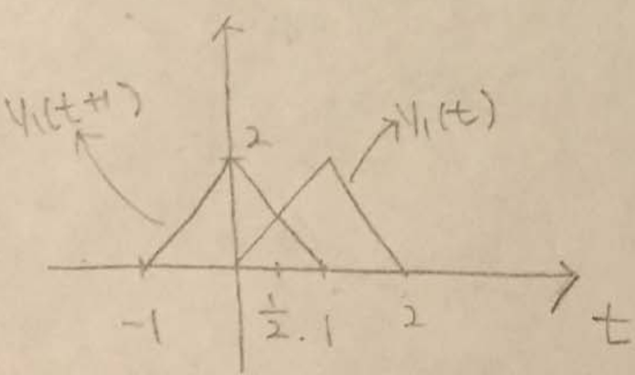


$$x_2(t) = x_1(t) - x_1(t-2)$$

$$\therefore y_2(t) = y_1(t) - y_1(t-2)$$

$$x_3(t) = x_1(t) + x_1(t+1)$$

$$\therefore y_3(t) = y_1(t) + y_1(t+1)$$



$\Rightarrow$