1. Can you find a sequence $\{a_n\}$ of real numbers where there are infinitely many n with $a_n > \limsup a_n$?

2. (a) Given a sequence $\{x_1, x_2, ...\}$ of real numbers, can you find a sequence, say, $\{a_n\}$ with $E = \{\text{limit points of } \{a_n\}\} = \{x_1, x_2, ...\}$?

   (b) Can you find a sequence $\{a_n\}$ of real numbers where $E = \{\text{limit points of } \{a_n\}\} = \mathbb{R}$?

3. Let $a_n, b_n \in \mathbb{R}$. Is it true that $\limsup a_n b_n = \limsup a_n \limsup b_n$? What if a_n and b_n are non-negative? Can you prove or disprove any inequality?

4. Rudin's definition of the limit supremum is

$$\limsup a_n = \sup E$$

   with E the set of limit points of $\{a_n\}$. Show this is equivalent to the following definition

$$\limsup a_n = \lim_{n\to\infty} \sup_{k\geq n} a_k.$$

   Hint: $\sup_{k\geq n} a_k$ decreasing in n. Remark: This second definition is often easier in practice.

5. Suppose $a_n > 0, a_n \to a, \lambda < a$ and show

$$\sum_0^\infty \lambda^n (a_0 a_1 \dots a_n)$$

   converges absolutely. (hint : compare to a geometric series).

6. In a metric space, show $\{a_n\}$ converges iff every subsequence $\{a_{n_k}\}$ has a further convergent subsequence $\{a_{n_{k_j}}\}$.

7. Suppose X is a complete metric space, and $\{G_n\}$ is a family of open sets, and each G_n is dense, i.e. $\bar{G_n} = X$. Show $\cap G_n$ is nonempty.