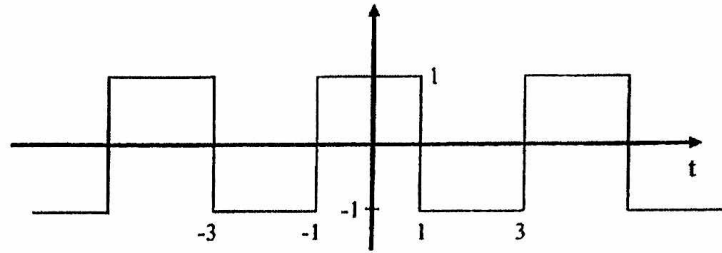
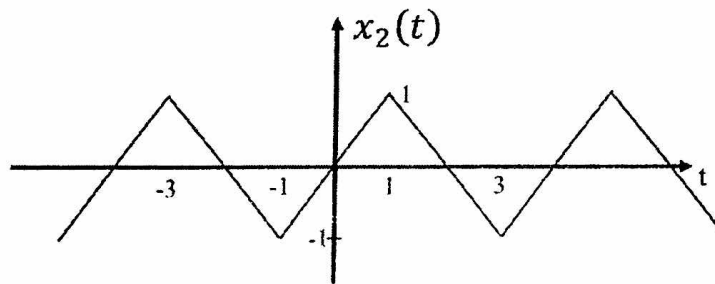


## 1. Fourier Series Calculations

Let  $x_1(t)$  be

- (a) (6 points) What is an appropriate period,  $T_0$ , for this signal?
- (b) (6 points) Using your  $T_0$ , set up the integral to find the Fourier series coefficients of  $x_1(t)$ ,  $a_k$ . You do not need to solve it, but you must have the correct integrand and limits.
- (c) (6 points) What would the Fourier series coefficients of  $x_2(t)$  be in terms of  $a_k$ ?



$$a) T_0 = 4$$

$$b) a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk \frac{2\pi}{T_0} t} dt$$

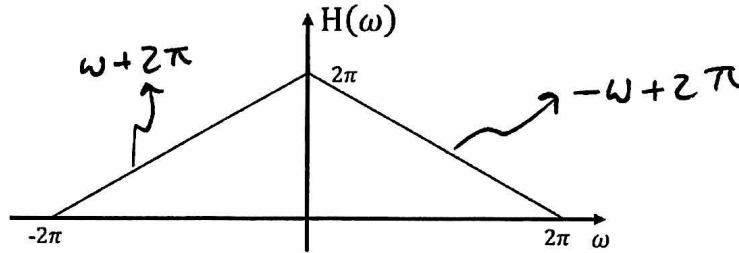
$$= \frac{1}{4} \int_{-1}^1 e^{-jk \frac{\pi}{2} t} dt - \frac{1}{4} \int_1^3 (-1) e^{-jk \frac{\pi}{2} t} dt$$

$$c) x_2(t) = \int_{-\infty}^t x_1(\tau) d\tau \Rightarrow x_2(t) \xleftrightarrow{FS} \frac{1}{jk(\frac{\pi}{2})} a_k$$

## 2. Fourier Series and LTI Systems

Let  $x(t) = 2e^{-j\frac{3\pi}{2}t} + 1e^{j\frac{\pi}{2}t} + 2e^{j\frac{3\pi}{2}t}$

- (a) (10 points) Find  $a_k$ , the Fourier series coefficients of  $x(t)$ , using the period  $T_0 = 4$ . Show all your steps.
- (b) (8 points) Let  $H(\omega)$  be as shown below.



Find the Fourier series coefficients of  $y(t) = x(t) * h(t)$ ,  $b_k$ .

If you did not find  $a_k$ , use  $a_k = \sin(k\pi/2)/(k\pi)$  and  $a_0 = 1/2$  instead.

- (c) (8 points) Give an expression for  $y(t)$ .

$$a) \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{\pi}{2}t}$$

$$= 2e^{-j\frac{3\pi}{2}t} + 1e^{j\frac{\pi}{2}t} + 2e^{j\frac{3\pi}{2}t}$$

By comparing  $x(t)$  with the synthesis equation:

$$a_{-3} = 2, \quad a_1 = 1, \quad a_3 = 2$$

all others are zero

$$b) \quad b_k = a_k H(\omega_k)$$

$$b_{-3} = a_{-3} H(-\frac{3\pi}{2}) = 2(-\frac{3\pi}{2} + 2\pi) = 2(\frac{\pi}{2}) = \pi$$

$$b_1 = a_1 H(\frac{\pi}{2}) = 1(-\frac{\pi}{2} + 2\pi) = \frac{3\pi}{2}$$

$$b_3 = a_3 H(\frac{3\pi}{2}) = 2(-\frac{3\pi}{2} + 2\pi) = 2(\frac{\pi}{2}) = \pi$$

all other  $b_k$  are zero

$$\begin{aligned} c) \quad y(t) &= \sum_{k=-\infty}^{\infty} b_k e^{jk\frac{\pi}{2}t} \\ &= \pi e^{-j\frac{3\pi}{2}t} + \frac{3\pi}{2} e^{j\frac{\pi}{2}t} + \pi e^{j\frac{3\pi}{2}t} \end{aligned}$$

### 3. Continuous Time Fourier Transform

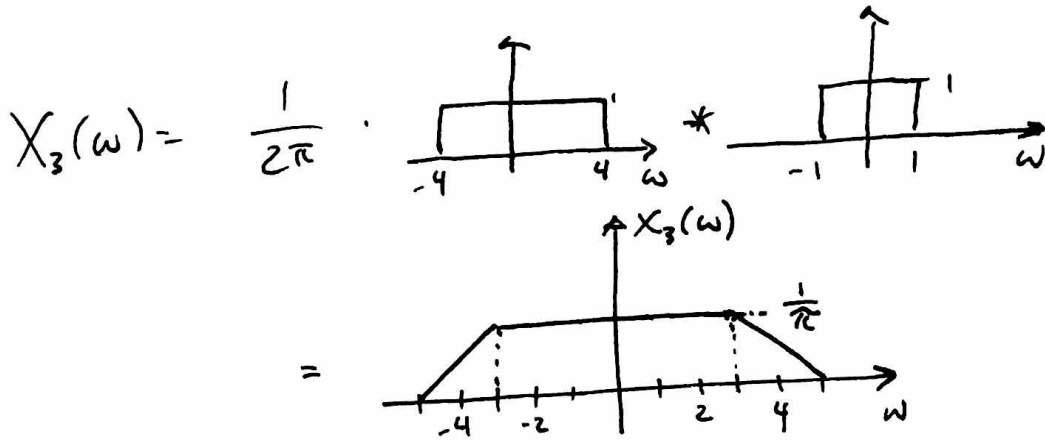
- (a) (6 points) Find  $X_1(\omega)$  given  $x_1(t) = e^{-6t}u(t)$  using the analysis equation. Show all your work.
- (b) (6 points) Find  $X_2(\omega)$  given  $x_2(t) = e^{-6|t|}$ . Use any method, but show your work.
- (c) (8 points) Give an expression for and plot  $X_3(\omega)$  given  $x_3(t) = \frac{\sin(4t)\sin(t)}{\pi^2 t^2}$ . Use any method, but show your work. Label key points on your plot.
- (d) (8 points) Give an expression for and plot  $X_4(\omega)$  given  $x_4(t) = \frac{\sin(4t)\sin(t)}{\pi^2 t}$ . Use any method, but show your work. Label key points on your plot.

$$\begin{aligned}
 \text{a) } X_1(\omega) &= \int_{-\infty}^{\infty} x_1(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-6t} e^{-j\omega t} dt \\
 &= \int_0^{\infty} e^{-t(6+j\omega)} dt = \frac{-1}{6+j\omega} \left[ e^{-t(6+j\omega)} \right]_0^{\infty} \\
 &= \frac{-1}{6+j\omega} \left[ e^{-\infty(6+j\omega)} - e^{-0(6+j\omega)} \right] = \frac{1}{6+j\omega}
 \end{aligned}$$

$$\text{b) } x_2(t) = x_1(t) + x_1(-t)$$

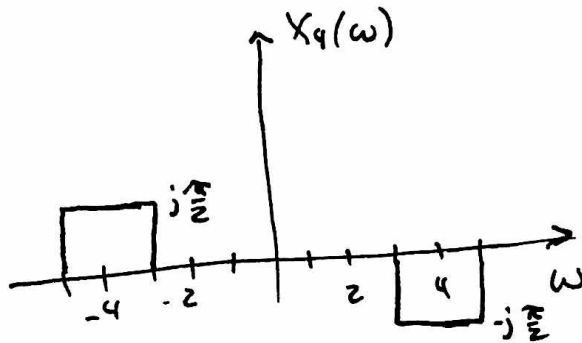
$$\begin{aligned}
 X_2(\omega) &= X_1(\omega) + X_1(-\omega) = \frac{1}{6+j\omega} + \frac{1}{6-j\omega} \\
 &= \frac{6-j\omega + 6+j\omega}{(6+j\omega)(6-j\omega)} = \frac{12}{36-\omega^2}
 \end{aligned}$$

$$c) x_3(t) = \frac{\sin 4t}{\pi t} \cdot \frac{\sin t}{\pi t}$$



$$X_3(\omega) = \begin{cases} \frac{\pi}{2}\omega + \frac{5\pi}{2}, & -5 < \omega < -3 \\ -\frac{\pi}{2}\omega + \frac{5\pi}{2}, & 3 < \omega < 5 \\ 1, & -3 < \omega < 3 \\ 0, & |\omega| > 5 \end{cases}$$

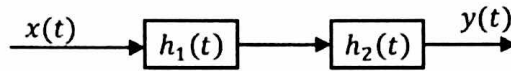
$$d) x_4(t) = t x_3(t) \Rightarrow X_4(\omega) = j \frac{d}{d\omega} X_3(\omega)$$



$$X_4(\omega) = \begin{cases} j \frac{\pi}{2}, & -5 < \omega < -3 \\ -j \frac{\pi}{2}, & 3 < \omega < 5 \\ 0, & \text{else} \end{cases}$$

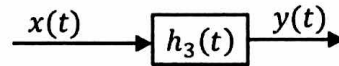
#### 4. CTFT and LTI Systems

Given the system



where  $h_1(t) = \frac{\sin(5t)}{\pi t} \cos(2t)$  and  $h_2(t) = \frac{\sin(7t)}{\pi t}$ .

(a) (6 points) Find an equivalent system with the impulse response  $h_3(t)$ .



(b) (6 points) Let  $x(t) = \sum_{k=1}^3 2^k \cos(4kt)$ . Find  $X(\omega)$ .

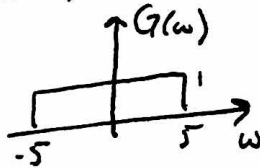
(c) (6 points) Find an expression for and plot  $Y(\omega)$ .

(d) (6 points) Find a simple expression for  $y(t)$ .

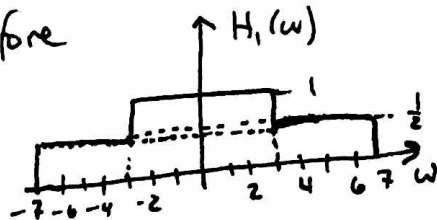
(e) (4 points) What restriction could you place on  $x(t)$  so that  $y(t) = x(t)$ ? In other words, what kind of signal will pass through the system unchanged.

a)  $H_1(\omega) = \frac{1}{2} (G(\omega-2) + G(\omega+2))$  by modulation,

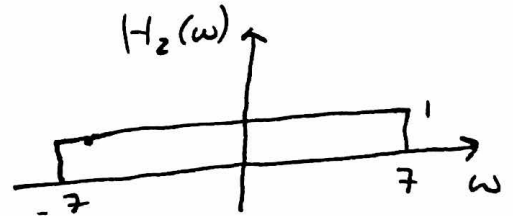
where



therefore



and



$$H_3(\omega) = H_1(\omega) H_2(\omega) = H_1(\omega)$$

$$\Rightarrow h_3(t) = h_1(t) = \frac{\sin 5t}{\pi t} \cos 2t$$

Equivalently,  $h_3(t) = \frac{1}{2} \frac{\sin 7t}{\pi t} + \frac{1}{2} \frac{\sin 3t}{\pi t}$

$$\begin{aligned}
 b) \quad X(\omega) &= \mathcal{F}\left\{ \sum_{k=1}^3 2^k \cos(4kt) \right\} = \sum_{k=1}^3 2^k \mathcal{F}\left\{ \cos(4kt) \right\} \\
 &= \sum_{k=1}^3 2^k \pi (\delta(\omega - 4k) + \delta(\omega + 4k)) \\
 &= 2\pi (\delta(\omega - 4) + \delta(\omega + 4)) + 4\pi (\delta(\omega - 8) + \delta(\omega + 8)) \\
 &\quad + 8\pi (\delta(\omega - 12) + \delta(\omega + 12))
 \end{aligned}$$

c)  $Y(\omega) = X(\omega) \cdot H_3(\omega)$

$Y(\omega) = \pi (\delta(\omega - 4) + \delta(\omega + 4))$

d) From Table 4.2  
 $y(t) = \cos(4t)$

e) If  $x(t)$  is bandlimited such that  $X(\omega) = 0$  for  $|\omega| \geq 3$ ,  
then  $y(t) = x(t)$ .