

1. Countable or uncountable (with proof)?
 - (a) $\oplus_{\mathbb{N}}\mathbb{Q} = \{(q_1, q_2, \dots) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \dots : \text{only finitely many } q_i \text{ are non-zero.}\}$.
 - (b) $\prod_{\mathbb{N}}\mathbb{Q} = \{(q_1, q_2, \dots) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \dots\}$
2. Let $f : A \rightarrow B$, A and B subsets of \mathbb{R}^n . True or false with proof:
 - (a) $\cup f^{-1}(A_j) = f^{-1}(\cup A_j)$
 - (b) $\cap f^{-1}(A_j) = f^{-1}(\cap A_j)$
3. Let A be a convex set in \mathbb{R}^n and show A° and \bar{A} are convex also.
4. (Lempert) Let A be an open, non-empty, convex set in \mathbb{R}^n , and show $A_r := \{x \in A : d(x, A^c) < r\}$ is also convex.
5. For any open $G \subseteq \mathbb{R}$, show G is a countable union of balls, i.e. $\exists \{B_{r_j}(x_j)\}_{j \in \mathbb{N}}$ with $G = \cup_j B_{r_j}(x_j)$.
6. True or False:

If (X, d) is a non-empty metric space, and $x \in X$ then $\overline{N_1(x)} = \{y : d(x, y) \leq 1\}$
7. Recall, if G_α are open, and C_α are closed, then we know $\cup G_\alpha$ is open and $\cap C_\alpha$ is closed.

Exhibit an example of open sets, G_α in say \mathbb{R}^n such that $\cap G_\alpha$ is not open.

Notice, by de Morgan's Law, we obtain a sequence C_α with $\cup C_\alpha$ not closed.
8. Consider \mathbb{Z} the integers as a subspace of \mathbb{R} . Let $A \subseteq \mathbb{Z}$. Is A open in \mathbb{Z} (i.e. is A open relative to \mathbb{Z} ?) Is A open in \mathbb{R} . What about closed relative to \mathbb{Z} ? \mathbb{R} ?