1. Let (X, d) be a metric space that satisfies the Heine-Borel property, namely that closed and bounded is equivalent to compact.

   (a) Let $K \subseteq X, C \subseteq X$ with K compact, C closed and $K \cap C = \emptyset$. Show there exists a compact $A \subseteq X$ with $K \subseteq A^o \subseteq A \subseteq C^c$.

   (b) For all $x \in X, B \subseteq X$ define

   $$d(x, B) = \inf\{d(x, b) : b \in B\}$$

   and show that if K is compact and nonempty in X, and $x \in X$ there is a $k \in K$ with $d(x, K) = d(x, k)$.

   Show this result holds if we replace K with any nonempty closed set.