Looking back

- Chap 1 : Signals
  - energy/power
  - periodicity
  - even/odd

  classifying signals

  Systems
  - memory
  - causality
  - stability (BIBO)
  - linearity
  - time invariance

  Key system properties to our analysis
- Chap 2 : LTI systems
  - represented the input as a sum of scaled and shifted deltas
  - found the output to a single delta, $\delta[n] / \delta(t)$
  - Superposition gave us convolution sum/integral

Fourier series representation of periodic signals

$$x(t) = \sum_k \underbrace{a_k}_{\text{scaling factor}} \underbrace{\phi_k(t)}_{\text{basis function}}$$

- We want two main properties from $\phi_k(t)$:
  - $\phi_k(t)$ to represent a large class of functions
  - analysis of LTI systems to be convenient given $\phi_k(t)$
- Eigenfunctions

  If $S\{x(t)\} = \lambda x(t)$

  $x(t)$ is an eigenfunction for the system S

  Key: the output is a scaled version of the input.

for LTI system

$$e^{st} \longrightarrow H(s)e^{st} \qquad s = \sigma + j\omega$$

$$z^{n} \longrightarrow H(z)z^{n} \qquad z \text{ is complex}$$

$$S\{e^{st}\} = \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}\,d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)e^{st}e^{-s\tau}\,d\tau = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}\,d\tau$$

$$= e^{st}\underbrace{\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}\,d\tau}_{H(s)}$$

$$= e^{st}H(s)$$

Ex $x(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_3 t}$

$$
\begin{aligned}
y(t) &= S\{x(t)\} \\
&= S\{a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_3 t}\} \\
&= a_1 S\{e^{s_1 t}\} + a_2 S\{e^{s_2 t}\} + a_3 S\{e^{s_3 t}\} \\
&= a_1 \underbrace{H(s_1)}_{H(s)\big|_{s=s_1}} e^{s_1 t} + a_2 H(s_2) e^{s_2 t} + a_3 H(s_3) e^{s_3 t}
\end{aligned}
$$

$$x(t) = \sum_k a_k e^{s_k t}$$

$$
\begin{aligned}
y(t) &= S\Big\{\sum_k a_k e^{s_k t}\Big\} = \sum_k a_k S\{e^{s_k t}\} \\
&= \sum_k a_k H(s_k) e^{s_k t}
\end{aligned}
$$

Notation:

$H(s) \sim$ Laplace $\qquad s = \sigma + j\omega$

$$H(s)\big|_{s=j\omega} = H(j\omega) \Rightarrow \underbrace{H(\omega),\ H(f)}_{\text{CTFT or DTFT}}$$

$H(z) \sim$ Z transform

$$H(z)\big|_{z=e^{j\omega}} = \underbrace{H(e^{j\omega})}_{\text{DTFT}}$$

Notation depends on context.

(A quick aside before moving on.)

Calculate FS coefficients

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

T_0 ~ fundamental period of $x(t)$

$$\underbrace{\int_0^{T_0} x(t) e^{-jn\omega_0 t}}_{\text{inner product}} dt = \int_0^{T_0} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^{T_0} e^{j(k-n)\omega_0 t} dt$$

$$\int_0^{T_0} e^{j(k-n)\omega_0 t} dt = \int_0^{T_0} \cos((k-n)\omega_0 t) dt + j \int_0^{T_0} \sin((k-n)\omega_0 t) dt$$

$$= \begin{cases} 0 & \text{if } |k-n| > 0 \\ \int_0^{T_0} e^{j0\omega_0 t} dt = T_0 & \text{if } k-n = 0 \\ & \quad k = n \end{cases}$$

$$\int_0^{T_0} x(t) e^{-jn\omega_0 t} dt = T_0 a_n \Rightarrow a_n = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jn\omega_0 t} dt$$

$$x(t) = \frac{1}{2} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \underbrace{\frac{1}{k\pi} \sin\left(\frac{\pi}{2}k\right)}_{a_k} e^{jk2\pi t}$$