

ECE 302 Homework 5

Due July 19, 2016

Reading assignment: chapter 5, sections 5.1 - 5.8.

1. The amplitudes of signals X and Y have joint pdf:

$$f_{X,Y}(x,y) = k(x+2y), \quad 0 < x < 1, 0 < y < 1,$$

where k is a constant.

- (a) Find k .
 - (b) Find the marginal pdfs.
 - (c) Find $\Pr(X < Y)$.
 - (d) Find $\Pr(X + Y \leq 1)$.
 - (e) Find the mean values of X and Y .
 - (f) Find the correlation and covariance of X and Y .
 - (g) Determine whether X and Y are independent, orthogonal, or uncorrelated.
2. Repeat Problem 1 with random variable X and Y with joint pdf:

$$f_{X,Y}(x,y) = kx(1-x)y, \quad 0 < x < 1, 0 < y < 1,$$

where k is a constant.

3. Consider the random variables X and Y with joint pdf as in Problem 1.
 - (a) Find $f_{Y|X}(y|x)$.
 - (b) Find $\Pr(Y > X|x)$.
 - (c) Find $\Pr(Y > X)$ using part b.
 - (d) Find $\mathbb{E}[Y|X]$.

4. Let X and Y be random variables with joint pdf:

$$f_{X,Y}(x,y) = ke^{-(x+y)}, \quad 0 < y < x < \infty,$$

where k is a constant.

- (a) Are X and Y independent? Explain why or why not.
 - (b) Find the value of k .
 - (c) Let $U = X + Y$. Find the pdf of U .
 - (d) Let $V = Y/X$. Find the pdf of V .
5. A student throws a dart at a dartboard. Let X and Y be independent random variables denoting the x-coordinate and y-coordinate, respectively, of the dart measured from the center of the dartboard. Assume that X and Y are both Gaussian random variables with mean 0 and variance σ^2 .
- (a) Find the probability that the distance from the center of the dartboard to the dart is greater than r , where $r \geq 0$. Hint: Use polar coordinates.
 - (b) Let R be the random variable denoting the distance from the center of the dartboard to the dart. Find the pdf of R . What is the name of the distribution of this random variable?