1. Let S be a set, and $\mathcal{P}(S) = \{\text{subsets of S}\}$. Do the relations "=" and "$\subset$" make $\mathcal{P}(S)$ into a totally orderded set?

2. Letting $\mathcal{P}(S)$ be as above, set $A+B := A \cup B$ and $AB := A \cap B$. Does this make $\mathcal{P}(S)$ a field?

3. Let A and B be nonempty subsets of $\mathbb{R}$. Define $A + B = \{a + b : a \in A, b \in B\}$. Show $\sup(A + B) = \sup A + \sup B$.

4. Let $A = \{x^n - x^m : 0 \leq x \leq 1 \text{ and non-negative integers } n, m\}$ and find $\sup A$.

5. Let $A \subset \mathbb{R}$ with $\emptyset \neq A$. Set $-A = \{-a : a \in A\}$ and show $-\sup A = \inf(-A)$.

6. Let $A \subset \mathbb{R}$ with $\emptyset \neq A$. Set $\alpha = \sup A$ and suppose $\alpha < \infty$. Also suppose there exists a $\delta > 0$ such that for all distinct a and b in A we have $|a - b| \geq \delta$. Show $\alpha \in A$.

7. Let $A \subset \mathbb{R}$ with $\emptyset \neq A$ and A bounded. Fix $x, y \in \mathbb{R}$. Set $T = \{ax + y : a \in A\}$ and find $\sup T$.