

ECE 302 Homework 7

Due August 2, 2016

Reading assignment: chapter 9, sections 9.1 - 9.4, 9.6; chapter 10, sections 10.1 - 10.3.

1. A coin is flipped n times. Let the random variable $X_i = 1$ if the i th flip is heads and $X_i = 0$ if the i th flip is tails, for $i = 1, 2, \dots, n$. Let X be the number of heads flipped in n flips. Assume all flips are fair and independent.
 - (a) What kind of random variable is X ? Express X as a function of X_1, X_2, \dots, X_n .
 - (b) Find the mean and variance of X .
2. A random process $X(t)$ is defined by

$$X(t) = \begin{cases} 1 & , T \leq t \leq T + 1, \\ 0 & , \text{else,} \end{cases}$$

where T is a uniformly distributed random variable in the interval $(0,1)$.

- (a) Plot a few sample functions of $X(t)$.
 - (b) Find the pmf of $X(t)$ for a fixed value of t .
 - (c) Find $\mu_X(t)$ and $R_X(t_1, t_2)$. Is $X(t)$ a wide-sense stationary random process?
3. A discrete-time random process is defined by $X(n) = A^n$, for $n \geq 0$. Assume A is a uniform random variable on the interval $(0, 1)$.
 - (a) Plot a few sample functions of $X(n)$.
 - (b) Find the pdf of $X(n)$ for a fixed value of n .
 - (c) Find $\mu_X(n)$ and $R_X(n_1, n_2)$. Is $X(n)$ a wide-sense stationary random process?

4. Students arrive at a train station according to a Poisson process with an arrival rate of 1 student per 5 minutes.
- (a) Find the probability that the first student will arrive in the first 10 minutes.
 - (b) Find the probability that the first two students will arrive in the first 10 minutes.
 - (c) Find the probability that no more than two students will arrive in the first 10 minutes.
5. The input into a filter is white Gaussian noise with noise power density $N_0/2$. The filter has transfer function

$$H(f) = \frac{1}{1 + j2\pi f}.$$

- (a) Find $S_{Y,X}(f)$ and $R_{Y,X}(\tau)$.
- (b) Find $S_Y(f)$ and $R_Y(\tau)$.
- (c) What is the average power of the output? What is the ratio between the output power and the input power?