

ECE 302 Homework 6

Due July 26, 2016

Reading assignment: chapter 5, sections 5.8, 5.9; chapter 6, section 6.5 .

1. Let X and Y be random variables denoting coordinates on the xy -plane. The rotation of the point (X, Y) through θ to the point (U, V) can be performed by letting:

$$\begin{aligned}U &= X \cos \theta - Y \sin \theta \\V &= X \sin \theta + Y \cos \theta\end{aligned}$$

Find $f_{U,V}(u, v)$.

2. Two lightbulbs from different brands have lifetimes X and Y that are independent and exponentially distributed with average lifetime $1/\lambda_1$ and $1/\lambda_2$, respectively. Both lightbulbs are turned on at the same time.
 - (a) Let U be the time elapsed until both lightbulbs have burned out. Find the pdf of U .
 - (b) Let V be the time elapsed until the first lightbulb has burned out. Find the pdf of V .
 - (c) Find the joint pdf of U and V .

3. Let X and Y be independent Gaussian random variables with mean 0 and variance 1. Let $U = aX + bY$ and $V = cX + dY$, where $ad - bd \neq 0$.

- (a) Are U and V jointly Gaussian? Explain why.
- (b) Find the mean, variance, and correlation coefficient of U and V .
- (c) Find the joint pdf of U and V .
- (d) Find the conditional mean and variance of U given V .
- (e) Find the conditional pdf of U given V .

4. Let X and Y be random variables with joint pdf:

$$f_{X,Y}(x, y) = \begin{cases} x + y & , 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & , \text{else} \end{cases}$$

- (a) Find the MAP estimator of X given $Y = y$.
- (b) Find the ML estimator of X given $Y = y$.
- (c) Find the MMSE estimator of X given $Y = y$.
- (d) Find the LMMSE estimator of X given $Y = y$.