1. Let $f : [0,1] \cup [2,3] \to \mathbb{R}$ continuous. If the image of f is connected, show f is not 1-1.

2. Let A and B be subsets of a metric space X.

   (a) Recall $d(x, A) := \inf_A d(x, a)$. If A is compact, show there is some $a \in A$ where the distance is obtained.

   (b) Suppose $X = \mathbb{R}^n$ and A is only assumed to be closed. Prove the result still holds.

   (c) Find a counter-example to show this is false in general when A is assumed only to be closed.

   (d) Now define $d(A, B) := \inf \{d(a, b) : a \in A,\ b \in B\}$. Show that if A and B are both compact there are $a \in A, b \in B$ for which the distance is obtained.

   (e) Can we relax this condition?