

$$5)(a) P(\sqrt{x^2+y^2} \geq r) = 1 - P(\sqrt{x^2+y^2} < r)$$

$$= 1 - \iint_{x^2+y^2 \leq r} f_{x,y}(x,y) dx dy$$

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) \quad \text{as } \mu_x = \mu_y = 0 \\ \sigma_x = \sigma_y = \sigma$$

$$\therefore P(\sqrt{x^2+y^2} \geq r) = 1 - \iint_{x^2+y^2 \leq r} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) dx dy$$

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ x &= r \cos \theta \\ y &= r \sin \theta \\ \theta &= \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}$$

$$= 1 - \frac{1}{2\pi\sigma^2} \int_0^{2\pi} \int_0^R \exp\left(-\frac{r^2}{2\sigma^2}\right) r dr d\theta$$

$$= e^{-R^2/2\sigma^2}$$

$$(b) F_R(r) = P(R \leq r) = 1 - e^{-r^2/2\sigma^2}$$

$$f_R(r) = \frac{d}{dr} F_R(r) = \frac{2r}{2\sigma^2} e^{-r^2/2\sigma^2}$$

$$= \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \quad \text{a Cauchy distribution}$$