Convergence and Absolute Convergence:

A series $\sum_{-\infty}^{\infty} a_n$ is called 'absolutely convergent' when $\sum_{-\infty}^{\infty} |a_n|$ converges

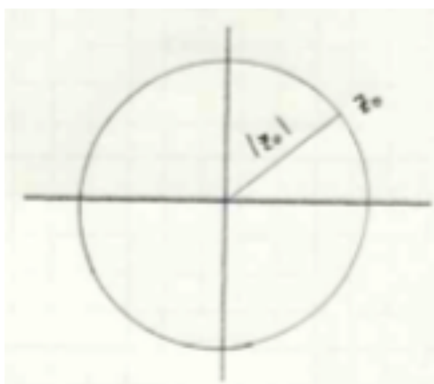
Fact about convergence:

- If $\sum_{-\infty}^{\infty} |a_n|$ converges, then $\sum_{-\infty}^{\infty} a_n$ converges also.
- $X(z)$ converges absolutely, if and only if $\sum_n |x[n]\, z^{-n}|$ converges

In literature and ECE 438, referring to Region of Convergence (R.O.C) is actually referring to Region of 'Absolute' Convergence

Facts about R.O.C.:

- R.O.C. is made up of rings around origin. If z_o is in the R.O.C., then any other z with $|z|>|z_o|$ is also in the R.O.C.
  - This relationship can be viewed in the sketch below:



- If $x[n]$ is 'causal' and z_o is in the R.O.C., then any z with $|z|>|z_o|$ is also in the R.O.C.
  - 'causal' → $x[n] = 0 \ \forall\, n < 0$

PROOF: for z with $|z|>|z_o|$

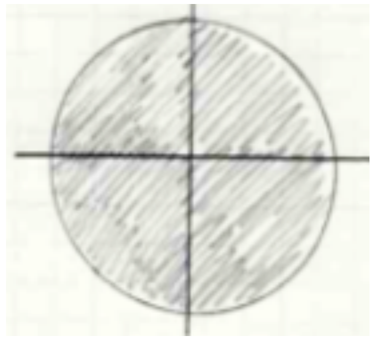
$\sum_{-\infty}^{\infty} |x[n]\, z^{-n}|$ = $\sum_{0}^{\infty} |x[n]\, z^{-n}|$

= $\sum_{0}^{\infty} |x[n]||z^{-n}|$ {where z > z_o; $|z|^{-n}$ < $|z_o|^{-n}$, since n > 0

$\leq \sum_{0}^{\infty} |x[n]||z_o{}^{-n}|$

= $\sum_{-\infty}^{\infty} |x[n]\, z_o{}^{-n}|$ ← converges by assumption; $\therefore X(z)$ converges absolutely

- If $x[n]$ is anti-causal and $z_o \in R.O.C$, then any z with $|z|<|z_o|$ is also in the R.O.C.
    - 'anti-causal' → $x[n] = 0 \;\; \forall > 0$
    - This relationship can be viewed in the sketch below:



- If $x[n]$ is mixed-causal (two sided signal), and $z_o \in R.O.C$ then there exists r_1, r_2 {$r_1 < |z_o| < r_2$; $r_1, r_2 \in$ R ≥ 0} such that $X(z)$ converges for all z with $r_1 < |z_o| < r_2$
    - This relationship can be viewed in the sketch below: