Formulas

CT

1.) Finite signal Energy

$$E = \int_{t_2}^{t_1} |x(t)|^2 \, dt$$

2.) Infinite signal Energy

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

3.) Finite signal Power

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

4.) Infinite signal Power

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

5.) Periodic signals.

$$x(t) \rightarrow x(t+T)$$

6.) Exponential signals

$$x(t) = Ce^{\alpha t}$$

$$C = |c| e^{j\phi}$$

$$\alpha = \sigma + j\omega$$

$$\therefore x(t) = |c| e^{\sigma t} e^{j(\omega t + \phi)}$$

DT

$$E = \sum_{n=N_1}^{N_2} |x[n]|^2$$

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$x[n] \rightarrow x[n+N]$$

$$x[n] = C\alpha^n$$

$$C = |c| e^{j\phi}$$

$$\alpha = |\alpha| e^{j\omega}$$

$$\therefore x[n] = |c| |\alpha|^n e^{j(\omega n + \phi)}$$

$$\{\phi = 0\}$$

7.) $\delta(t)$ & $u(t)$

(i) $x(t)\,\delta(t) = x(0)\,\delta(t)$

(ii) $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

$\delta[n]$ & $u[n]$

$\delta[n] = u[n] - u[n-1]$

$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

$x[n]\,\delta[n] = x[0]\,\delta[n]$

$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$

8.) Unit Step

$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$

9.) Unit Impulse

$\delta(t) = d/dt\, u(t)$

$\delta(mt) = \frac{1}{m}\,\delta(t)$

$\delta'(-t) = -\delta'(t)$

$\delta[mn] = \delta[n]$

10.) Convolution sum / Integral

$x(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

$x(t) * \delta(t) = x(t)$

$x(t) * \delta(t-t_0) = x(t_0)$

11.) Memory less of an LTI

$h(t) = A\delta(t)$

$y(t) = Ax(t)$

$h[n] = A\delta[n]$

$y[n] = Ax[n]$

12.) Invertibility of an LTI

$$h(t) * h_{inv}(t) = A\delta(t)$$

Similar

$$h[n-k] = 0 \quad k > n$$
$$h[n] = 0 \quad k < 0$$

13.) Causality -

$$h(t-\tau) = 0 \qquad \tau > t$$
$$h(t) = 0 \quad \text{for } t < 0$$

14.) Stability -

$|y(t)| < \infty$ iff

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

$|y[n]| < \infty$ iff

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

15.) Step Response

$$s(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau)\, d\tau$$
$$= \int_{0}^{\infty} h(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} h(\tau)\, d\tau$$

$$s[n] = \sum_{k=-\infty}^{\infty} h[k]$$

$$s[n] = s[n-1] = h[n]$$

$$\frac{d}{dt}[s(t)] = h(t)$$

16.) Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

17.) Analysis Eq. -

$$a_k = \frac{1}{T} \int x(t) e^{-jk\omega_0 t}\, dt$$

$$a_0 = \frac{1}{T} \int_T x(t)\, dt$$

Formulas (Contd.)

1.) **Time Shifting -**

$$x(t) \longrightarrow x(t - t_0)$$

(a) $t_0 > 0$ (delay)

(b) $t_0 < 0$ (advance)

2.) **Time Scaling -**

$$x(t) \longrightarrow x(\alpha t)$$

(a) $\alpha > 1$ (faster) (shrink)

(b) $\alpha < 1$ (slower) (signal expands)

3.) **Time Reversal -**

$$x(t) \longrightarrow x(-t)$$

4.) **Even signals -**

$$x(t) = x(-t) \quad \boxed{\text{OR}} \quad x[n] = x[-n]$$

5.) **Odd signals**

$$x(t) = -x(-t)$$

6.) **Even & Odd Decomposition -**

$$\mathcal{E}v\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

$$Od\{x(t)\} = \frac{[x(t) - x(-t)]}{2}$$

7) Fundamental Period -

$$\frac{K}{N} = \frac{\omega}{2\pi}$$

8.) Linearity

(a) - Additivity - $x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$

b) - Homogeneity - $ax_1(t) \xrightarrow{S} a y_1(t)$

$\therefore$ $x_1(t) \xrightarrow{S} y_1(t)$

$x_2(t) \xrightarrow{S} y_2(t)$

$ax_1(t) + bx_2(t) \xrightarrow{S} ay_1(t) + by_2(t)$

9.) Time Invariance -

$x_1(t) \xrightarrow{S} y_1(t)$

$x_2(t) = x_1(t - t_0) \xrightarrow{S} y_2(t) = y_1(t - t_0)$

10.) Domain chain in Integrals -

$$\int_{\alpha}^{\beta} x(t)\,dt = \int_{\alpha+T}^{\beta+T} x(t)\,dt$$

iff

$$x(t) = x(t+T)$$

11.) Equalities

(a) $\sum_{n=0}^{N-1} a^n = \begin{cases} \frac{1-a^N}{1-a} & \text{for } a \neq 1 \\ N & \text{for } a = 1 \end{cases}$

(b) $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$

(c) $\sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a} \qquad |a| < 1$

(d) $\sum_{n=0}^{\infty} n a^n = \frac{1}{(1-a)^2} \qquad |a| < 1$

12.) Integral of unit impulse

$$\int_a^b \phi(t)\,\delta(t)\,dt = \begin{cases} \phi(0) & a < 0 < b \\ 0 & a < b < 0 \text{ or } 0 < a < b \\ \text{undefined} & a = 0 \text{ or } b = 0 \end{cases}$$

13.) LTI Response to complex exponentials -

$x(t) = e^{st}$

$y(t) = e^{st} H(s)$

$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$

$y[n] = z^n H(z)$

$x[n] = z^n$

$H(z) = \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}$

Random Formula's

1.) $e^{\pm 2\pi j} = 1$

2.) $\omega = 2\pi f$ OR $f = \left[\frac{2\pi}{\omega}\right]^{-1}$

3.) $T = \frac{2\pi}{\omega}$

4.) $|e^{j(\omega t+\Phi)}|^2 = 1$

5.) $e^{j\omega N} = 1$

6.) $e^{j\omega_0 N_0 T_s} = 1$

$\omega_0 N_0 T_s = m2\pi$

7.) $e^{-j\pi} = -1$

8.) $b_k = \frac{a_k}{j4\,2\pi/T}$

9.) $\int_a^b uv\,dw = uv\Big|_a^b - \int_a^b v\,dw$

10.) $\cos \alpha t = \frac{1}{2}(e^{\alpha t} + e^{-\alpha t})$

11.) $\sin \alpha t = \frac{1}{2j}(e^{\alpha t} - e^{-\alpha t})$

12.) $\sin(\omega t) = \cos(\omega t - \pi/2) = -\cos(\omega t + \pi/2)$

$\cos(\omega t) =$ ____

13.) $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$