" $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ "

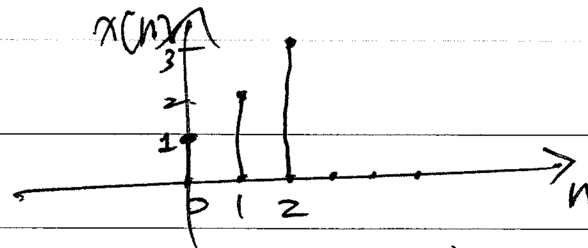
2) Three ways to compute DT convolution

- Method 1 : collectively sum

$y[n] = \cdots + x[-1]h[n+1] + x[0]h[n] + x[1]h[n-1] + \cdots$

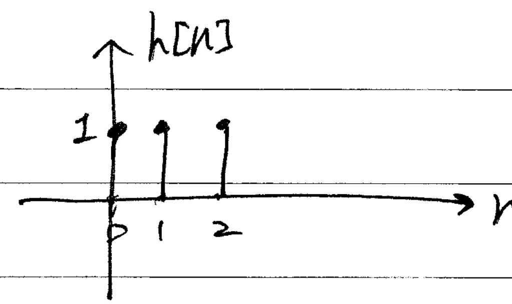
e.g. $y[n] = x[n] + x[n-1] + x[n-2]$

Find output when $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$



- Impulse response of system?

$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$



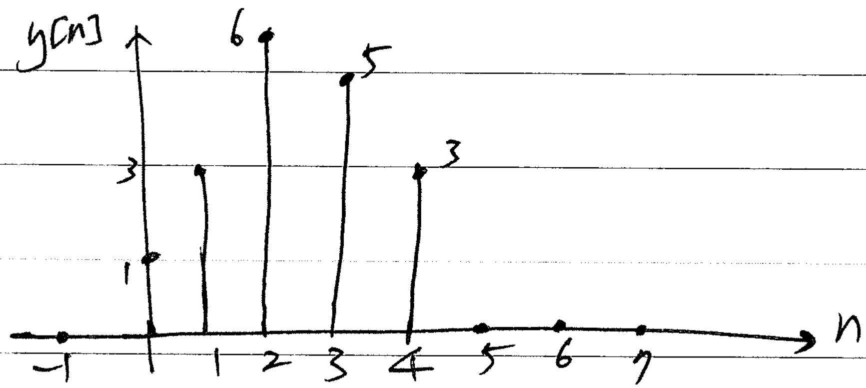
- Since $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$,

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$= \underbrace{x[0]}_{1}h[n] + \underbrace{x[1]}_{2}h[n-1] + \underbrace{x[2]}_{3}h[n-2]$ $\left(\begin{array}{l} x[n],\ n \geq 3 \\ = 0 \end{array}\right)$

$= h[n] + 2h[n-1] + 3h[n-2]$ $\left(\begin{array}{l} x[n],\ n < 0 \\ = 0 \end{array}\right)$

$$y[n] = (\delta[n] + \delta[n-1] + \delta[n-2])$$
$$+ 2(\delta[n-1] + \delta[n-2] + \delta[n-3])$$
$$+ 3(\delta[n-2] + \delta[n-3] + \delta[n-4])$$



* $x[n]$ of length 3, $h[n]$ of length 3 $\rightarrow$ $x[n] * h[n]$ of length $3+3-1=5$

$\rightarrow$ Generally $x[n]$ of length N_1, $h[n]$ of length N_2 $\rightarrow$ $x[n] * h[n]$ of length $N_1 + N_2 - 1$

Note : not concerned with initial conditions in this course.
~~unless stated otherwise assume system is initially at rest.~~
~~$\Rightarrow$ all initial conditions $= 0$~~

- Method 2 : "run" input signal thru. difference equation

(Note : all DT LTI systems may be expressed as a difference equation). $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

e.g. (previous. one) " $x[n]=0$, $\begin{cases} n<0 \\ n>2 \end{cases}$. $x[0]=1$, $x[1]=2$, $x[2]=3$ "

$y[0] = x[0] + x[-1] + x[-2] = 1 + 0 + 0 = 1$

$y[1] = x[1] + x[0] + x[-1] = 2 + 1 + 0 = 3$

$y[2] = x[2] + x[1] + x[0] = 3 + 2 + 1 = 6$

$y[3] = x[3] + x[2] + x[1] = 0 + 3 + 2 = 5$

$y[4] = x[4] + x[3] + x[2] = 0 + 0 + 3 = 3$

$y[n] = 0$, for $n > 4$

* thru : through

- Method 3 : Graphical Method similar to that for CT conv.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

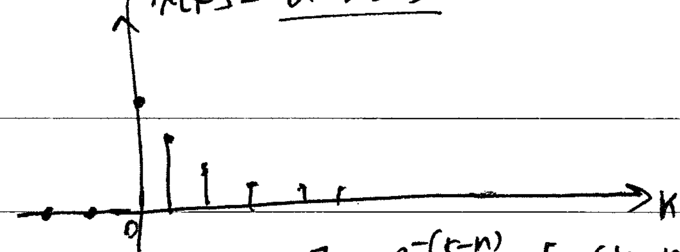
1. View $x[k]$ and $h[n-k] = h[-(k-n)]$ as functions of k
2. Flip $h[k]$ about $k=0$ to form $h[-k]$
3. Time-shift $h[-k]$ to the right by n to form $h[-(k-n)]$
4. Pointwise-multiply to form product $x[k]\, h[-(k-n)]$
5. Sum the values of the product (over all k)
6. Repeat for each value of n.

e.g. (Method 3) $x[n] = \alpha^n u[n]$, $h[n] = \beta^n u[n]$ $(\alpha \neq \beta)$

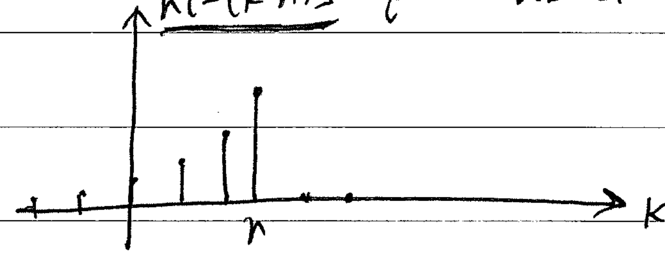
$y[n] = 0$ for $n < 0$

for $n > 0$,

$x[k] = \alpha^k u[k]$



$h[-(k-n)] = \beta^{-(k-n)} u[-(k-n)]$



$$y[n] = \sum_{k=0}^{n} \alpha^k \beta^{n-k} = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k$$

$$= \beta^n \cdot \frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\left(\frac{\alpha}{\beta}\right)} = \beta^n \cdot \frac{\beta - \frac{\alpha^n \alpha}{\beta^n}}{\beta - \alpha}$$

$$= \left(\frac{\beta}{\beta-\alpha}\beta^n - \frac{\alpha}{\beta-\alpha}\alpha^n\right) u[n]$$

since starts at $n=0$

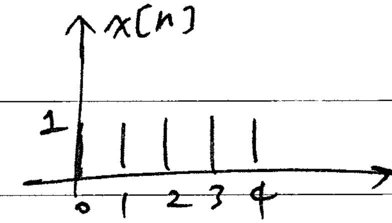
* recall $e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}(e^{at} - e^{bt})\, u(t)$

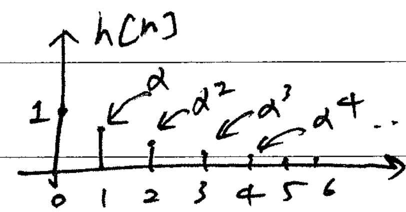
: CT conv.

e.g. (Method 1) Example 2.4 in text

$x[n] = u[n] - u[n-5]$

$h[n] = \alpha^n \{u[n] - u[n-7]\}$





$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

n	0	1	2	3	4	5	6	7	8	9	10	11	...
$x[0]h[n]$	1	α	α^2	α^3	α^4	α^5	α^6	0	0	0	0	0	
$x[1]h[n-1]$	0	1	α	α^2	α^3	α^4	α^5	α^6	0	0	0	0	
$x[2]h[n-2]$	0	0	1	α	α^2	α^3	α^4	α^5	α^6	0	0	0	
$x[3]h[n-3]$	0	0	0	1	α	α^2	α^3	α^4	α^5	α^6	0	0	
$x[4]h[n-4]$	0	0	0	0	1	α	α^2	α^3	α^4	α^5	α^6	0	
$y[n]$	1	$1+\alpha$	$1+\alpha+\alpha^2$	$\frac{1-\alpha^3}{1-\alpha}$	$\frac{1-\alpha^4}{1-\alpha}$	......				$\alpha^5+\alpha^6$	α^6		

(can apply method 2 ← textbook approach)