1. Let (X, d) be a metric space.

   (a) Let $K \subseteq X, C \subseteq X$ with K compact, C closed and $K \cap C = \emptyset$. Show there exists and open G in X with $K \subseteq G \subseteq C^c$.

   (b) Suppose now that closed balls in X are compact, (for all $x \in X, r > 0$, we suppose $\{y : |x - y| \leq r\}$ is compact.) Using the notation above, show there exists an open $G \subseteq X$ such that $\bar{G}$ is compact and $K \subseteq G \subseteq \bar{G} \subseteq C^c$

   (c) For all $x \in X, A \subseteq X$ define

   $$d(x, A) = \inf\{d(x, a) : a \in A\}$$

   and show that if K is compact and nonempty in X, and $x \in X$ there is a $k \in K$ with $d(x, K) = d(x, k)$.

   If $X = \mathbb{R}^n$ with the usual (Euclidean metric) show the above result holds if we replace K with any nonempty closed set.