1. (a) Can you find a metric space, say (X, d) which has a non-empty proper subset A which is both closed and open in X? (Hint: it suffices to consider $\mathbb{R}^n$.)

   (b) Can you find a metric space as above that is connected?

2. Let (X, d) be a metric space that satisfies the Heine-Borel property, namely that closed and bounded is equivalent to compact.

   (a) Let $K \subseteq X, C \subseteq X$ with K compact, C closed and $K \cap C = \emptyset$. Show there exists a compact $A \subseteq X$ with $K \subseteq A^o \subseteq A \subseteq C^c$.

   (b) For all $x \in X, B \subseteq X$ define

   $$d(x, B) = \inf\{d(x, b) : b \in B\}$$

   and show that if K is compact and nonempty in X, and $x \in X$ there is a $k \in K$ with $d(x, K) = d(x, k)$.

   Show this result holds if we replace K with any nonempty closed set.

3. Recall the Cantor set, C, is the set in $[0, 1]$ that is left after we repeatedly throw away the middle open intervals of length $\frac{1}{3}$. Show $[0, 1] - C$ is dense in $[0, 1]$.

4. Repeat the construction of the Cantor set, but this time cut out the middle open interval of length $\frac{1}{4}$. Is the remaining set open? closed? compact? perfect? Is its compliment dense?