12 April 2002 **Name:___________________**

EE 438 **Exam No. 3** **Spring 2002**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (30 pts.)

a. A wide-sense stationary process $X[n]$ with mean 1 and autocorrelation function

$$r_{XX}[n] = \begin{cases} 1, & n = 0 \\ 0.5, & n = \pm 1 \\ 0, & \text{else} \end{cases}$$

is input to the system described by $y[n] = x[n] - x[n-1]$.

Find the mean $\mu_Y[n]$ and autocorrelation $r_{YY}[n]$ of the output process $Y[n]$.

b. The same wide-sense stationary process $X[n]$ as in part a. above is input to the system described by $y[n] = x[2n]$.

Find the mean $\mu_Y[n]$ and autocorrelation $r_{YY}[n]$ of the output process $Y[n]$.

c. The same wide-sense stationary process $X[n]$ as in parts a. and b. above is input to the system described by

$$y[n] = \begin{cases} x[n/2], & n/2 \text{ is an integer} \\ 0, & \text{else} \end{cases}$$

Find the mean $\mu_Y[n]$ and autocorrelation $r_{YY}[n]$ of the output process $Y[n]$.

1. (continued)

2. (20 pts) An individual with pitch frequency 50 Hz utters a voiced phoneme with a strong first formant at 250 Hz and a weaker second formant at 1 kHz.

   a. Sketch the wideband and narrowband spectrograms corresponding to this speech waveform. Be sure to label all important quantities.

   b. Design a digital system consisting of a pulse generator driving a linear filter to synthesize this waveform. The system operates at an 8 kHz sampling rate. For this system, specify the pulse interval in samples and the approximate location in the Z plane of the poles for the filter.

2. (continued)

3 (25 pts.) In class, we defined the STDTFT of the signal $x[n]$ as

$$X(\omega, n) = \sum_k x[k]w[n-k]e^{-j\omega k}.$$

Suppose that we evaluate the STDTFT at L points $\omega_l = 2\pi l / L, l = 0, \ldots, L-1$ along the frequency axis; so

$$X[l, n] = \sum_k x[k]w[n-k]e^{-j2\pi lk/L}$$

a. Show that for each fixed value of l, $X[l,n]$ can be viewed as the output of a narrowband filter. Find an expression for the frequency response of this filter, and sketch what it would typically look like.

In class, we also showed that $x[n]$ could be reconstructed by summing the outputs of these L filters, provided the impulse responses $h_l[n]$ of the filters all had value $1/L$ at $n = 0$, and value 0 at $n = \pm L, \pm 2L, \pm 3L, \ldots$.

b. Show that a filter with frequency response

$$H(\omega) = \begin{cases} \frac{1}{L}\left(1 - |\omega / (2\pi / L)|\right), & |\omega| < 2\pi / L \\ 0, & \text{else} \end{cases}$$

satisfies this condition.

3. (continued)

4. (25 pts.) Your grades on quizzes 1, 2, and 3 are 3, 4, and 7, respectively. Based on this data set, find the least squares prediction $\hat{g}[4]$ for your grade on quiz 4, where the predictor has the form $\hat{g}[n] = a_0 + a_1 n$. Here n denotes the index of the quiz; and the coefficients a_0 and a_1 are chosen to minimize

$$E = \sum_{n=1}^{3} \left| \hat{g}[n] - g[n] \right|^2 ,$$

where $g[n]$ is the grade for the n-th quiz.

4. (continued)

1.	________
2.	________
3.	________
4.	________
Total	________