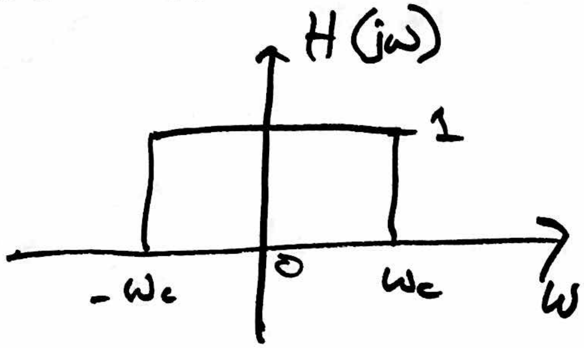
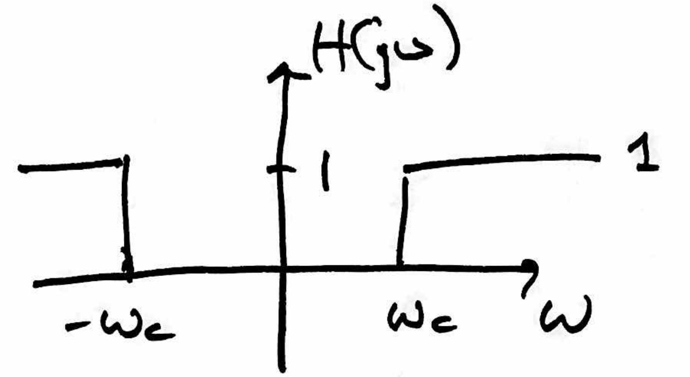
Filtering

- Basic Filters

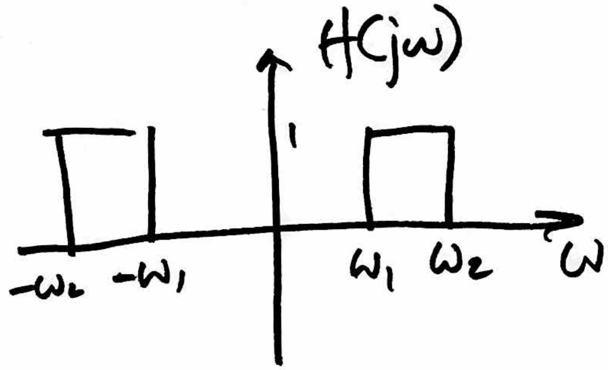
CT:



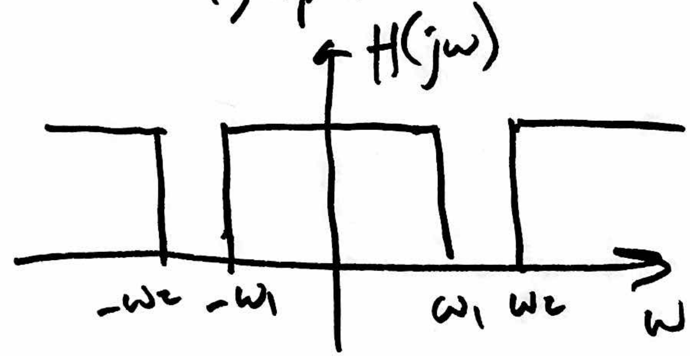
lowpass



highpass



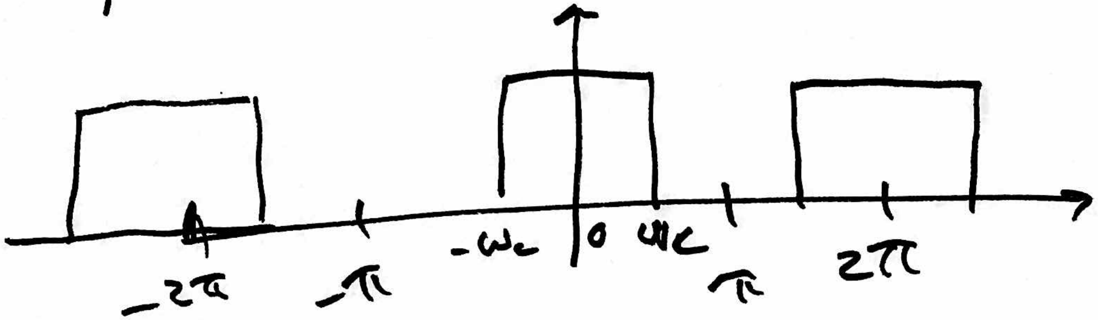
Bandpass



Bandstop

DT: You have the same filters, but they are periodic with period 2π.

DT LP filter



<u>Ex</u> $y[n] = \frac{1}{2}(x[n] + x[n-1])$

causal, two-point moving average

Find frequency response, $H(e^{j\omega})$.

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$$

$$x[n] = \delta[n] \Rightarrow y[n] = h[n]$$

$$h[n] = \frac{1}{2}(\delta[n] + \delta[n-1])$$

$$H(e^{j\omega}) = \frac{1}{2} e^{-j\omega 0} + \frac{1}{2} e^{-j\omega 1} = \frac{1}{2} + \frac{1}{2} e^{-j\omega}$$

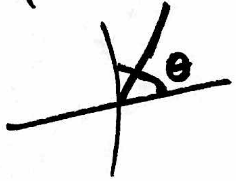
$$= e^{-j\frac{\omega}{2}}\left(\frac{e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}}{2}\right) = e^{-j\frac{\omega}{2}} \cos\left(\frac{\omega}{2}\right)$$

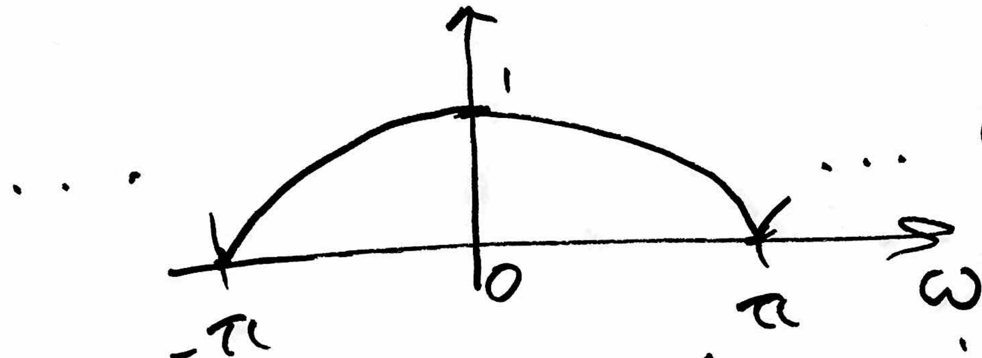
Plot $|H(e^{j\omega})| = |e^{-j\frac{\omega}{2}}\cos(\frac{\omega}{2})| = |e^{-j\frac{\omega}{2}}||\cos(\frac{\omega}{2})|$

$= \cos(\frac{\omega}{2})$ $\quad -\pi < \omega < \pi$

positive in our ω range

$|e^{j\theta}| = 1$

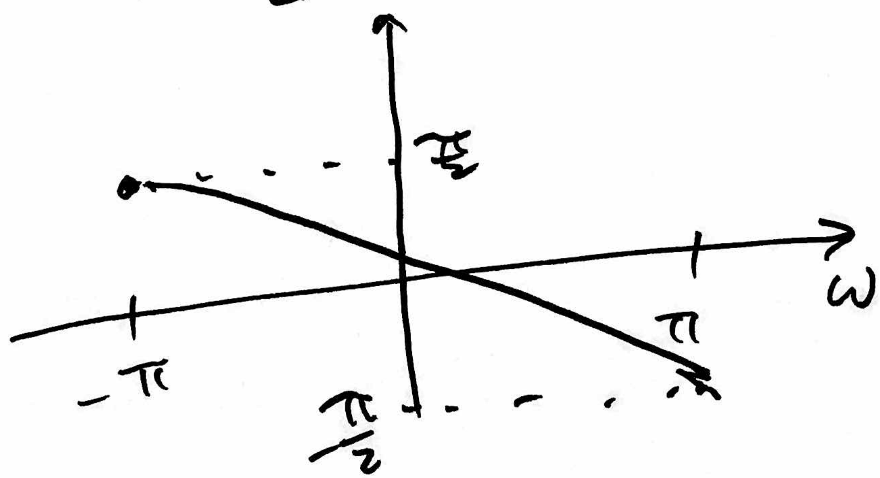




periodic with period 2π

$\angle H(e^{j\omega}) = \angle(e^{-j\frac{\omega}{2}}\cos\frac{\omega}{2}) = \angle e^{-j\frac{\omega}{2}} + \angle\cos\frac{\omega}{2}$

$= -\frac{\omega}{2} + 0 = -\frac{\omega}{2}$



Ex Recursive Equation

$$y[n] - a\,y[n-1] = x[n]$$

$$x[n] = e^{j\omega n} \qquad y[n] = H(e^{j\omega})e^{j\omega n}$$

$$H(e^{j\omega})e^{j\omega n} - aH(e^{j\omega})e^{j\omega(n-1)} = e^{j\omega n}$$

$$H(e^{j\omega}) - aH(e^{j\omega})e^{-j\omega} = 1$$

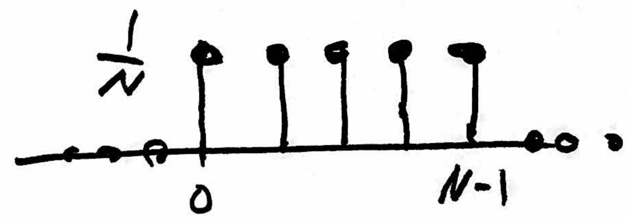
$$H(e^{j\omega})(1 - ae^{-j\omega}) = 1$$

$$H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$$

Ex Moving average (causal)

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k]$$

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1} \delta[n-k]$$



Find frequency response, $H(e^{j\omega})$

$$H(e^{j\omega}) = \sum_{n} h[n]e^{-j\omega n} = \sum_{k=0}^{N-1} \frac{1}{N} e^{-j\omega n}$$

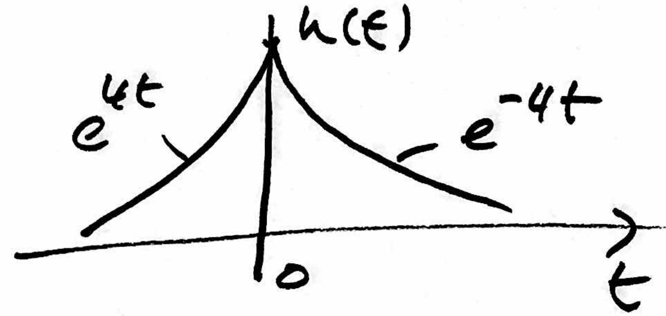
$1 - e^{j\theta}$
$= e^{-j\frac{\theta}{2}}(e^{j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}})$

$$= \frac{1}{N}\,\frac{1-e^{-j\omega N}}{1-e^{-j\omega}} = \frac{1}{N}\,\frac{e^{-j\frac{\omega}{2}N}}{e^{-j\frac{\omega}{2}}}\,\frac{e^{j\frac{\omega N}{2}} - e^{-j\frac{\omega}{2}N}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}$$

$$= \frac{1}{N}\, e^{-j\frac{\omega}{2}(N-1)}\,\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

Ex

$$h(t) = e^{-4|t|}$$



First, find $H(j\omega)$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t}\,dt$$

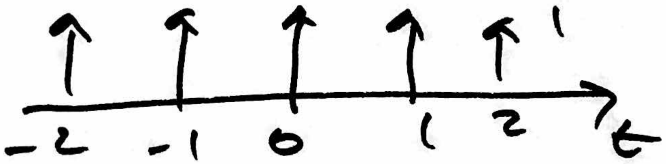
$$= \int_{-\infty}^{0} e^{4t} e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{-4t} e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{t(4-j\omega)}\,dt + \int_{0}^{\infty} e^{-t(4+j\omega)}\,dt$$

$$= \frac{1}{4-j\omega}\underbrace{\left[e^{t(4-j\omega)}\right]_{-\infty}^{0}}_{1-0} + \frac{-1}{4+j\omega}\underbrace{\left[e^{-t(4+j\omega)}\right]_{0}^{\infty}}_{0-1}$$

$$= \frac{1}{4-j\omega} + \frac{1}{4+j\omega}$$

a) $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$



$T_0 = 1$

Fin $x(t) \overset{FS}{\longleftrightarrow} a_k$

$y(t) = x(t) * h(t) \longleftrightarrow b_k$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-j\omega_0 k t}\, dt = \int_{-0.5}^{0.5} \delta(t) e^{-j\omega_0 k t}\, dt$$

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$

$$= \int_{-0.5}^{0.5} \delta(t)\, dt = 1$$

$\omega_0 = \frac{2\pi}{1} = 2\pi$

$a_k = 1$ for all k

$$b_k = H(j\omega_0 k) a_k = \frac{1}{4-j2\pi k} + \frac{1}{4+j2\pi k}$$

$$H(j\omega) = \Big|_{\omega = 2\pi k}$$