Wide-Sense Stationary R.P.s

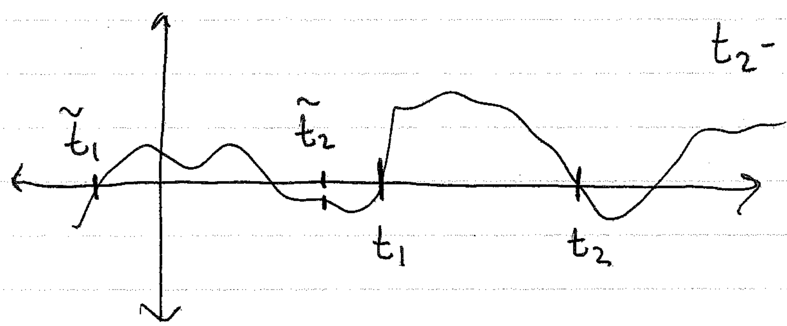
The mean value function of $X(t)$

is $\mu_X(t) = E[X(t)]$ (function of time)

The autocorrelation function of $X(t)$ is

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)]$$

$t_2 - t_1 = \tilde{t}_2 - \tilde{t}_1$



Let $X(t)$ be a r.p.

$X(t)$ is wide-sense stationary (WSS) if

1) $\mu_X(t) = \mu_X = \text{constant}$

2) $R_X(t_1, t_2) = R_X(t_2 - t_1) \leftarrow$ function $t_2 - t_1$

$\quad = R_X(\tau) \quad , \tau = t_2 - t_1$

If $X(t)$ is WSS

$$\Rightarrow R_X(t_1, t_2) = R_X(\tilde{t}_1, \tilde{t}_2) = R_X(\tau)$$

Properties of Autocorrelation Function of WSS r.p.s

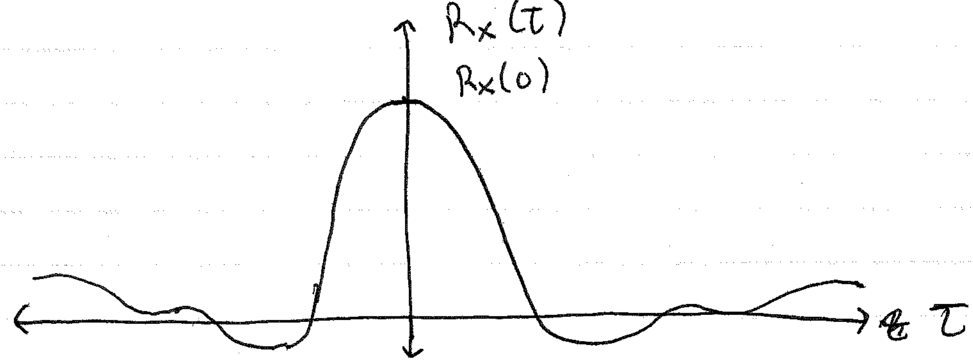
Let $X(t)$ be a WSS r.p. with $R_X(\tau)$

1) $E[X^2(t)] = R_X(0) \geq 0$

2) $E[X(t_1)X(t_2)] = R_X(t_2 - t_1) = R_X(t_1 - t_2)$

3) $R_X(\tau) = R_X(-\tau)$ (even)

4) $|R_X(\tau)| \leq R_X(0)$



Ex 1) Constant r.p.

$X(t) = A \quad , \quad -\infty < t < \infty$

where A is a r.v.

$\mu_X(t) = E[X(t)] = E[A] = \text{constant}$

$R_X(t_1, t_2) = E[X(t_1)X(t_2)]$

$= E[A^2] = \text{function of } t_2 - t_1$

$\cancel{R_X(t)} \quad R_X(\tau) = E[A^2]$

$\therefore$ $X(t)$ is WSS

2) Cosine with random amplitude

$$X(t) = A\cos(\omega t), \quad -\infty < t < \infty$$

where A is a r.v. and ω ~~is~~ > 0.

$$\mu_X(t) = E[X(t)]$$
$$= E[A\cos(\omega t)]$$
$$= E[A]\cos(\omega t)$$

$$R_X(t_1, t_2) = E[X(t_1)\,X(t_2)]$$
$$= E[A\cos(\omega t_1)\,A\cos(\omega t_2)]$$
$$= E[A^2]\cos(\omega t_1)\cos(\omega t_2)$$

$X(t)$ is WSS if $\mu_X(t)$ is a constant and $R_X(t_1, t_2) = R_X(t_2 - t_1) = R_X(\tau)$

$X(t)$ is WSS if

1) $E[A] = 0$

2) $E[A^2] = 0$