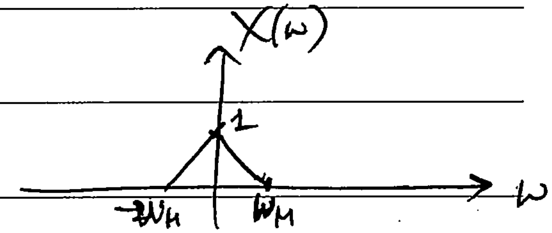
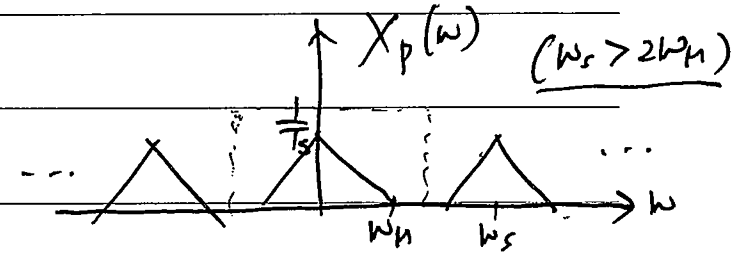
$$p(t) = \sum_n \delta(t - nT_s)$$

$$x(t) \longrightarrow \otimes \xrightarrow{x_p(t)} \boxed{\begin{matrix}\text{LPF} \\ H(\omega)\end{matrix}} \longrightarrow x_r(t)$$

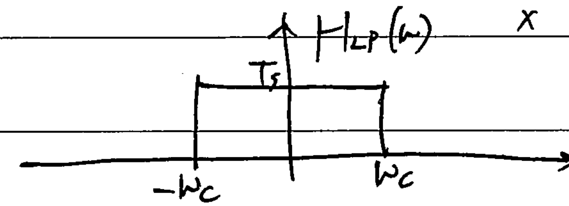


- Initially consider Ideal LPF with cut-off at $\omega_c = \frac{\omega_s}{2}$



$$h_{LP}(t) = T_s \frac{\sin\left(\frac{\omega_s}{2} t\right)}{\pi t} \qquad \omega_s = \frac{2\pi}{T_s}$$

$$= T_s \frac{\sin\left(\frac{\pi}{T_s} t\right)}{\pi t}$$



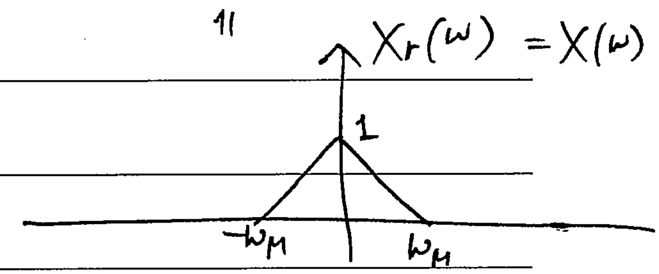
- reconstructed signal

$$x_r(t) = x_p(t) * h_{LP}(t)$$

$$= \left\{ \sum_n x(nT_s)\, \delta(t - nT_s) \right\} * h_{LP}(t)$$

$$= \sum_n x(nT_s)\, h_{LP}(t - nT_s)$$

$$= \sum_n x(nT_s) \frac{\sin\left(\frac{\pi}{T_s}(t - nT_s)\right)}{\frac{\pi}{T_s}(t - nT_s)}$$



· Thus, if $\omega_s = \frac{2\pi}{T_s} > 2\omega_M$ $\left(\text{or} \quad F_s = \frac{1}{T_s} > 2 f_{max}\right)$ (Hz)

sampling rate $\qquad$ period.

then:

$$\underline{x(t)} = \underline{x_r(t)} = \sum_n x(nT_s) \frac{\sin\left(\frac{\pi}{T_s}(t-nT_s)\right)}{\frac{\pi}{T_s}(t-nT_s)}$$

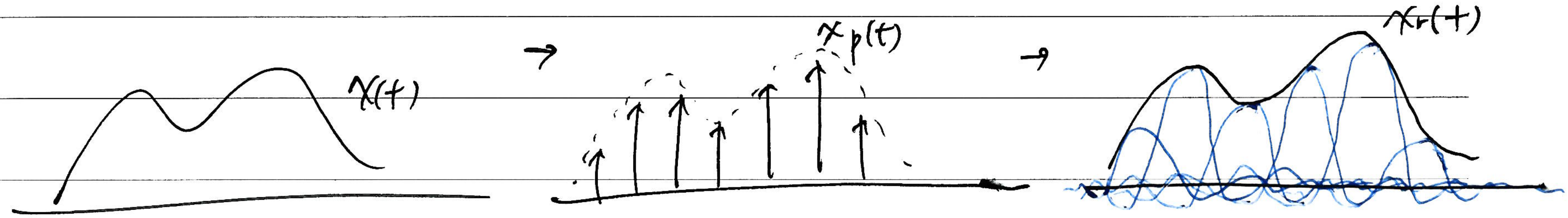
$$= \sum_n x[n] \left\{ \frac{\sin\left(\frac{\pi}{T_s}(t-nT_s)\right)}{\frac{\pi}{T_s}(t-nT_s)} \right\}$$

Just numbers.

(values of analog signal at equi-spaced instants in time)

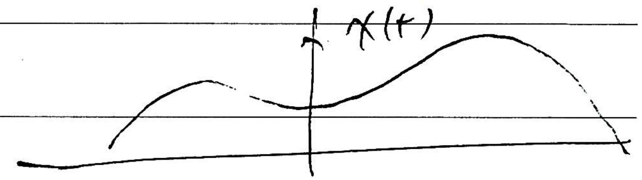
sinc function.

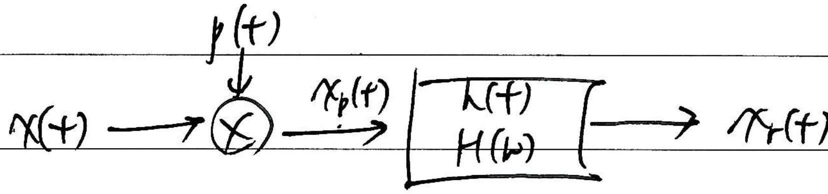
(interpolating func.)

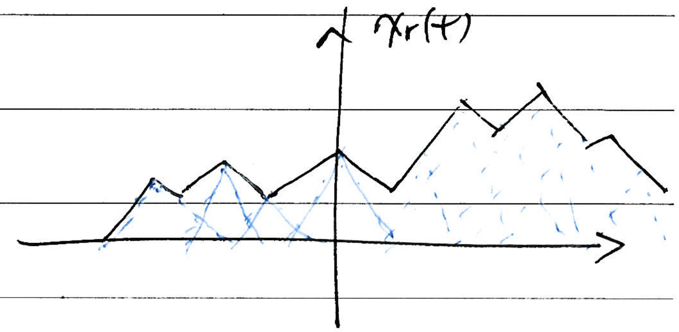


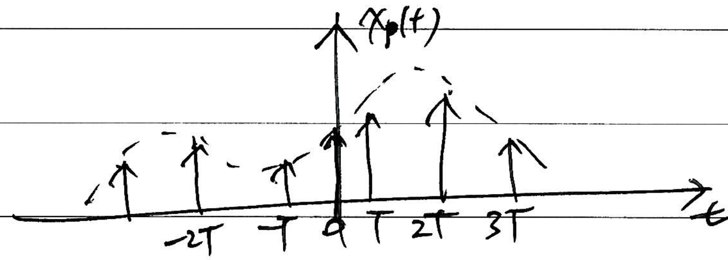
- Consider simpler interpolating functions.

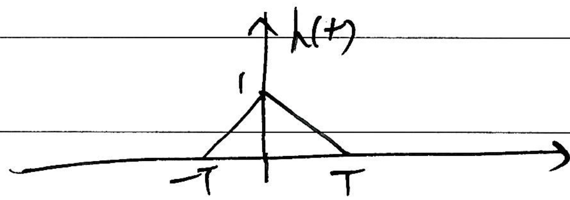
(eg. linear interpolation)

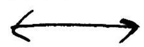


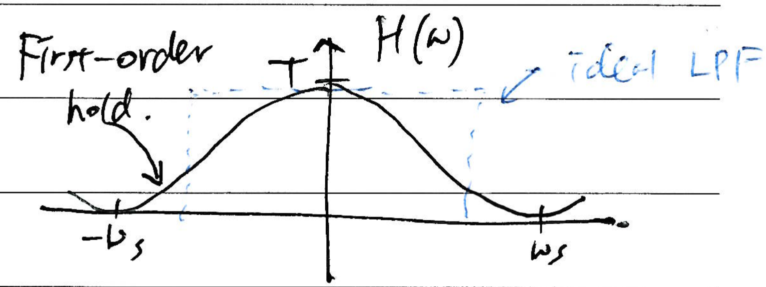






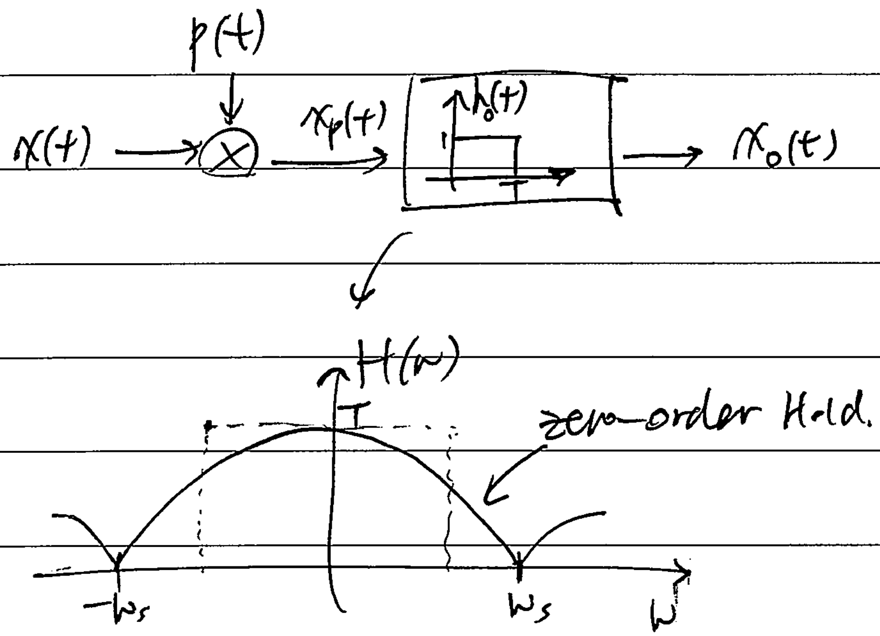


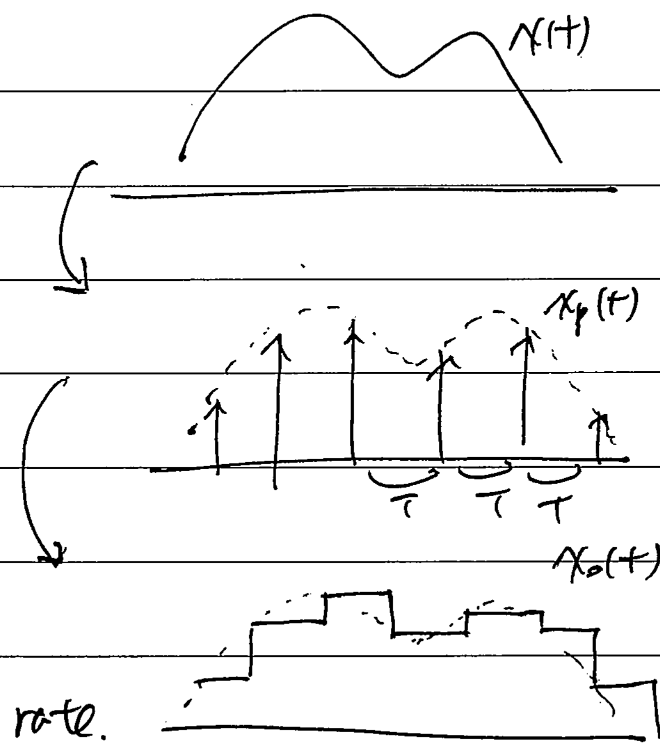




- will work if the initial sampling rate was significantly greater than the Nyquist rate.

- Simplest Scheme : zero-order Hold.
  (Charge a capacitor to current sample voltage value and hold constant until next sample time.)





- works well if sampling rate is substantially greater than Nyquist rate.

  sampling rate ↑ ⇒ T ↓

- Sec. 7.5.2 describes a way to increase the sampling rate digitally just prior to D/A conversion.

* Further Results on Sampling Theory and Ideal DAC.

- Recall FT pair

$$x_s(t) = \sum_n x_a(nT_s)\,\delta(t-nT_s) \xleftrightarrow{\mathcal{F}} X_s(\omega) = F_s \sum_k X_a(\omega - k\omega_s)$$

where : $\omega_s = 2\pi \underset{(Hz)}{F_s}$ and $F_s = \frac{1}{T_s}$ "sampling rate"

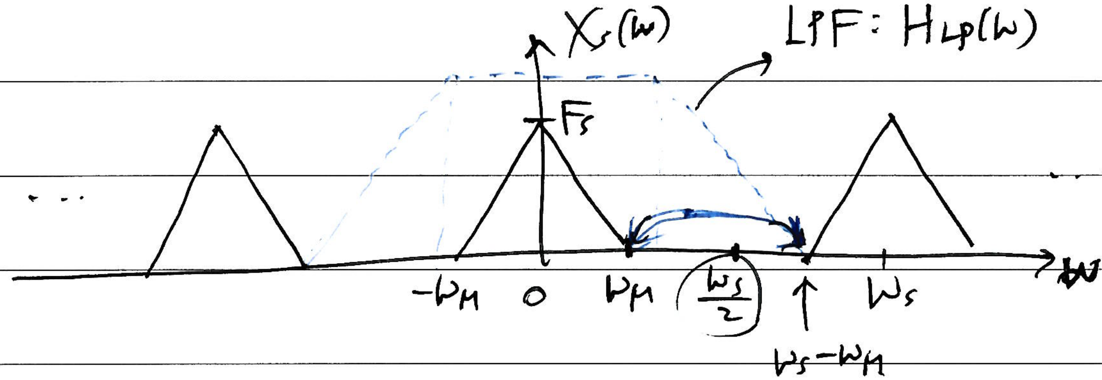
$$x_a(t) \xleftrightarrow{\mathcal{F}} X_a(\omega)$$

- Sampling Theory assumes some max freq. ω_M for which $X(\omega) = 0$ for $|\omega| > \omega_M$.

ω_M is referred to as the bandwidth for a baseband signal.

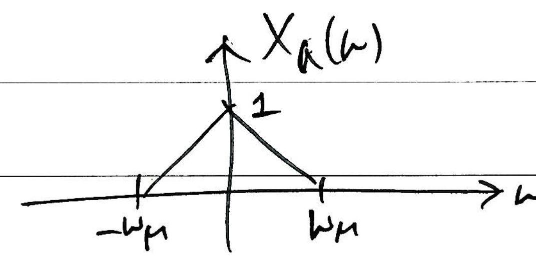
* ADC : $A \to D$ (sampling)
  DAC : $D \to A$ (reconstruction)

- We have standard picture for triangular spectrum



assuming: $\omega_s > 2\omega_M$ $(\omega_s - \omega_M > \omega_M)$

and $X_a(\omega) = 0$ for $|\omega| > \omega_M$



$\frac{\omega_s}{2} = \frac{1}{2}\{\omega_M + (\omega_s - \omega_M)\}$: middle of "don't care" gap.

that results with greater than Nyquist rate sampling.