More CTFT properties

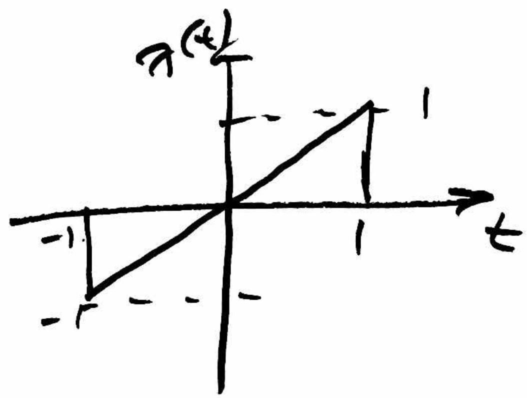
- Differentiation and integration

$$x(t) \xleftrightarrow{CTFT} X(\omega)$$

$$\frac{d}{dt}x(t) \xleftrightarrow{CTFT} j\omega X(\omega)$$

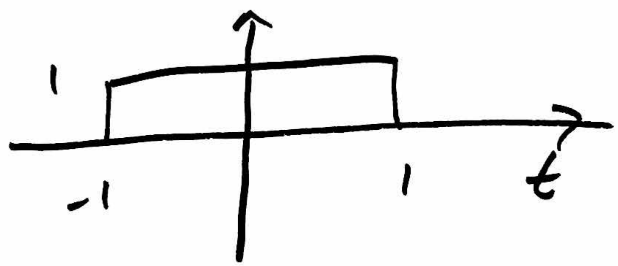
$$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{CTFT} \frac{1}{j\omega}X(\omega) + \underbrace{\pi X(0)\,\delta(\omega)}_{\text{handles a DC offset}}$$

<u>Ex</u>



find $X(\omega)$

$g(t) = x'(t)$



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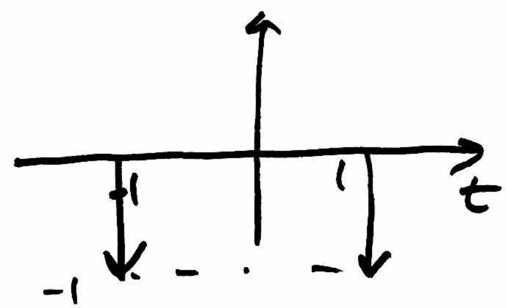


take care of discontinuities

Find $G(\omega)$

$$g(t) = \underbrace{\mathrm{rect}\left(\tfrac{t}{2}\right)}_{a} + \underbrace{\left(-\delta(t+1) - \delta(t-1)\right)}_{b}$$

a

$$\mathrm{rect}(t) \xleftrightarrow{\text{CTFT}} \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$x(at) \xleftrightarrow{\text{CTFT}} \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\Rightarrow a = \frac{1}{2}$$

$$\mathrm{rect}\left(\frac{t}{2}\right) \longleftrightarrow 2 \cdot \mathrm{sinc}\left(\frac{\omega}{\pi}\right)$$

b

$$\int_{-\infty}^{\infty} \left(-\delta(t+1)\right) e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} \left(-\delta(t-1)\right) e^{-j\omega t}\, dt$$

$$= -e^{+j\omega} \int_{-\infty}^{\infty} \delta(t+1)\, dt - e^{-j\omega} \int_{-\infty}^{\infty} \delta(t-1)\, dt$$

$$= -e^{j\omega} - e^{-j\omega} = -2\cos\omega$$

$$G(\omega) = 2\,\mathrm{sinc}\left(\frac{\omega}{\cancel{2}\pi}\right) - 2\cos\omega$$

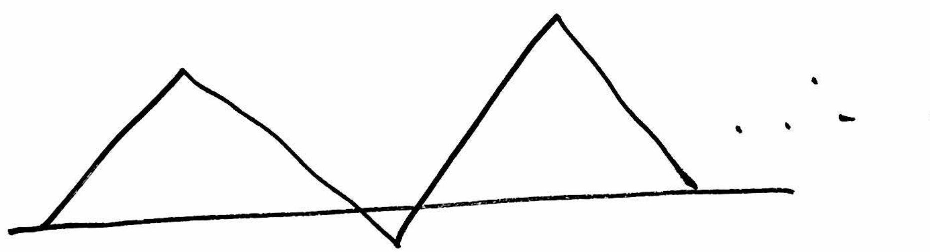
$$X(\omega) = \frac{1}{j\omega} G(\omega) + \pi G(0)\, \delta(\omega)$$

$$= \frac{2}{j\omega} \operatorname{sinc}\left(\frac{\omega}{\pi}\right) + \pi G(0)\, \delta(\omega) \qquad - 2\cos\omega$$

$$G(0) = 2 \operatorname{sinc}(0) - 2\cos 0$$

$$= 2 \cdot 1 - 2 \cdot 1 = 0$$

$$X(\omega) = \frac{2}{j\omega} \operatorname{sinc}\left(\frac{\omega}{\pi}\right) - \frac{2\cos\omega}{j\omega}$$



no discontinuities