Kushagra ...

Commutative Property:

CT: $x(t) * h(t) = h(t) * x(t)$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{+\infty}^{-\infty} x(t-u)\, h(u)\,(-du)$$

$$= (-)\int_{+\infty}^{\infty} x(t-u)\, h(u)\, du$$

$$= (-)(-)\int_{-\infty}^{\infty} x(t-u)\, h(u)\, du$$

$$\int_{-\infty}^{\infty} h(u)\, x(t-u)\, du = h(t) * x(t)$$

$u = t - \tau$
$\tau = t - u$
$d\tau = -du$
$\tau \to -\infty,\ u \to \infty$
$\tau \to \infty,\ u \to -\infty$

Hence Proved

DT: $x[n] * h[n] = h[n] * x[n]$

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$\Rightarrow \sum_{z=\infty}^{-\infty} x[n-z]\, h[z]$$

$$\sum_{z=-\infty}^{\infty} h[z]\, x[n-z]$$

$$= h[n] * x[n]$$ Hence Proved.

$z = n - k$
$k \to -\infty,\ z \to \infty$
$k \to \infty,\ z \to -\infty$

Distributive Property

CT: $x(t) * [h_1(t) + h_2(t)] = x(t) * z(t)$

$z(t) = h_1(t) + h_2(t)$

$$x(t) * z(t) = \int_{-\infty}^{\infty} x(\tau)\, z(t-\tau)\, d\tau$$

$z(t-\tau) = h_1(t-\tau) + h_2(t-\tau)$

$$\int_{-\infty}^{\infty} x(\tau) \cdot \left[h_1(t-\tau) + h_2(t-\tau) \right] d\tau$$

$$\int_{-\infty}^{\infty} x(\tau)\, h_1(t-\tau)\, d\tau + \int_{-\infty}^{\infty} x(\tau)\, h_2(t-\tau)\, d\tau$$

$$= x(t) * h_1(t) + x(t) * h_2(t)$$

Hence Proved

DT: $x[n] * (h_1[n] + h_2[n])$

Let $h_1[n] + h_2[n] = z[n]$

$$x[n] * z[n] = \sum_{k=-\infty}^{\infty} x[k]\, z[n-k]$$

$$z[n-k] = h_1[n-k] + h_2[n-k]$$

$$\sum_{k=-\infty}^{\infty} x[k] \cdot (h_1[n-k] + h_2[n-k])$$

$$\sum_{k=-\infty}^{\infty} (x[k] \cdot h_1[n-k]) + \sum_{k=-\infty}^{\infty} x[k] \cdot h_2[n-k]$$

$$= x[n] * h_1[n] + x[n] * h_2[n]$$

Hence Proved.

Associative Property:

CT: $x(t) * (h_1(t) * h_2(t))$

$$= x(t) * \int_{-\infty}^{\infty} h_1(\tau) h_2(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} x(z) \left(\int_{-\infty}^{\infty} h_1(\tau) h_2(t-\tau-z) d\tau \right) dz$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(z) h_1(\tau) h_2(t-\tau-z)\, d\tau\, dz$$

$\tau + z = m$, $d\tau = dm$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(z) h_1(m-z) h_2(t-m)\, dz\, dm$$

$$\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \left[x(z) h_1(m-z) dz \right] h_2(t-m) dm \right.$$

$$= \int_{-\infty}^{\infty} \left[x(m) * h_1(m) \right] \cdot h_2(t-m)\, dm$$

$$\Rightarrow [x(t) * h_1(t)] * h_2(t)$$ Hence Proved.

DT: $x[n] * \{ h_1[n] * h_2[n] \}$

$$= x[n] * \left\{ \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k] \right\}$$

$$= \sum_{l=-\infty}^{\infty} x[l] \left\{ \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k-l] \right\}$$

$$\sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[l] h_1[k] h_2[n-k-l]$$

$l + k = q$

$$\sum_{l=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} x[l] h_1[q-l] h_2[n-q]$$

$$\sum_{q=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} x[l] h_1[q-l] h_2[n-q]$$

$$\sum_{q=-\infty}^{\infty} \left\{ \sum_{l=-\infty}^{\infty} x[l] h_1[q-l] \right\} h_2[n-q]$$

$$\sum_{q=-\infty}^{\infty} \left\{ x[q] * h[q] \right\} h_2[n-q]$$

$$\left\{ x[n] * h_1[n] \right\} * h_2[n]$$

Hence Proved