"exists" "such that"

If $\exists\, T \neq 0$ s.t $x(t+T) = x(t)$, then $x(t)$ is periodic.

If $\exists$ smallest $T > 0$ s.t. $\forall t$, $x(t+T) = x(t)$, then T is the fundamental period.

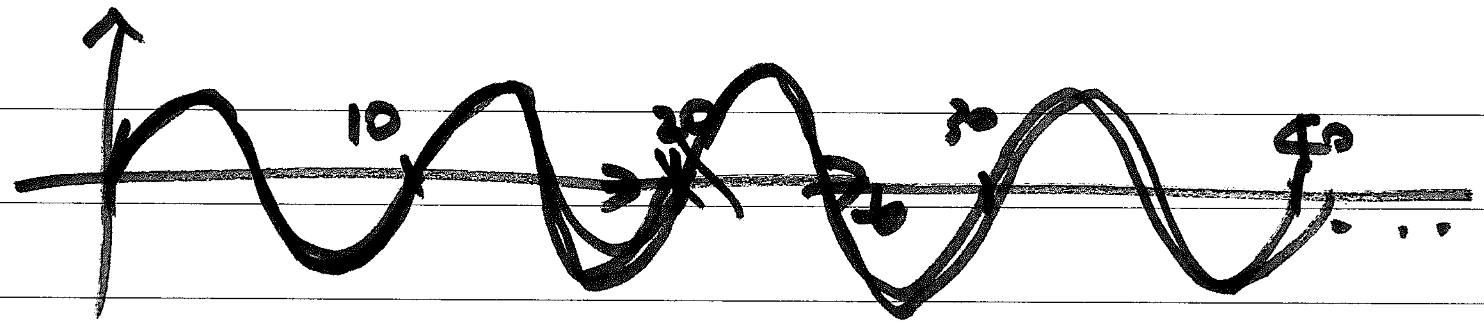
→ divide out <u>greatest common divisor</u> (gcd) between m and N

(from yesterday) $\frac{\omega_0}{2\pi} = \frac{m}{N}$ ← integers

$$e^{j\omega_0 n} = e^{j 2\pi \frac{m}{N}(n+N)} = e^{j 2\pi \frac{m}{N} n}$$

→ resulting period is called the fundamental period $N_0 = \frac{N}{\gcd(m,N)}$

e.g.



period : 10 ?

: $\underline{20}$?

fundamental period : 10 ?

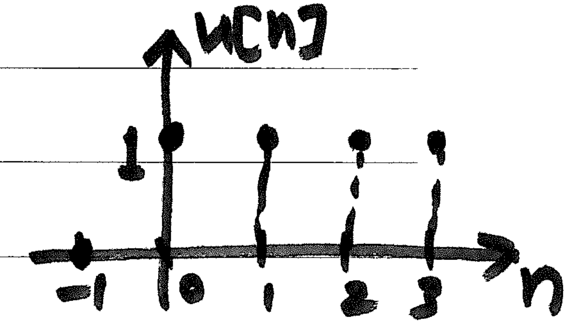
20 ?

- Table 1.1 in textbook summarizes differences btw $\underset{(CT)}{\underline{e^{j\omega_0 t}}}$ and $\underset{(DT)}{\underline{e^{j\omega_0 n}}}$

* btw : between

3) Basic DT signals

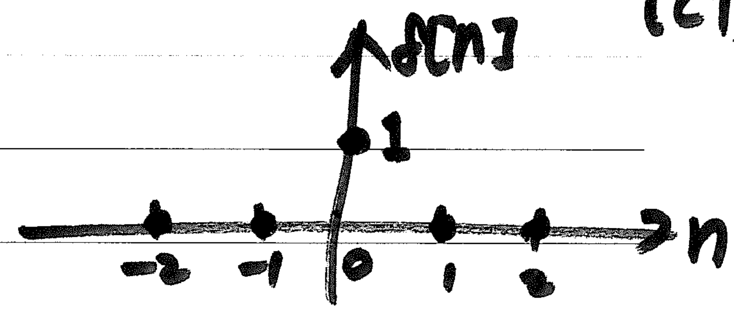
- unit step : $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



- Kronecker Delta Function ↔ Dirac Delta Func (CT)

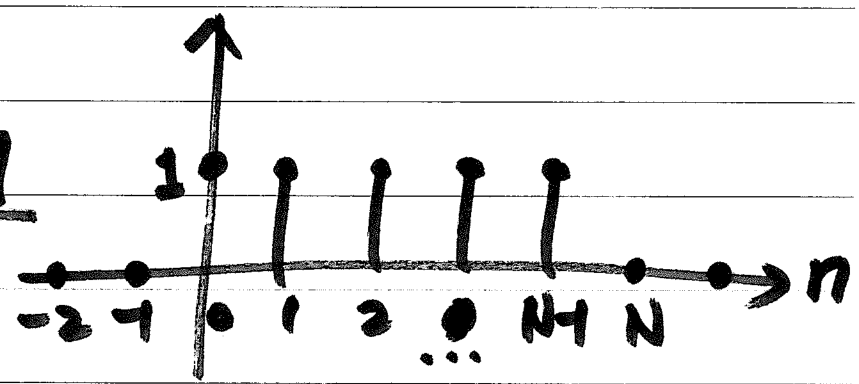
  (DT impulses)

  : $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$



- DT rectangle

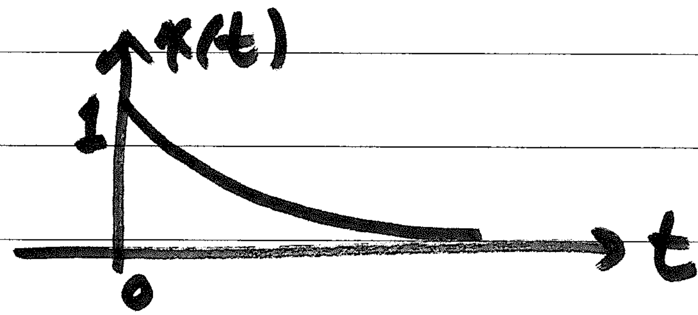
  : $u[n] - u[n-N]$



4) CT Exponential Signals and
DT Geometric Signals (sequences)

- CT : $x(t) = e^{-at}u(t)$
  - , where a can be complex-valued (in general)
  - , $u(t)$: unit step

- If a is real-valued and $a > 0$

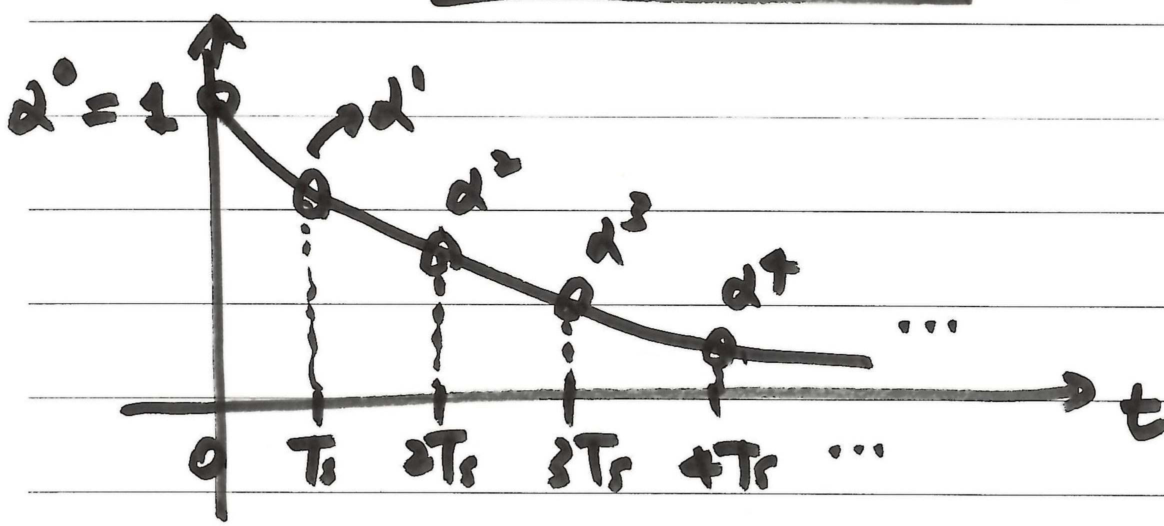


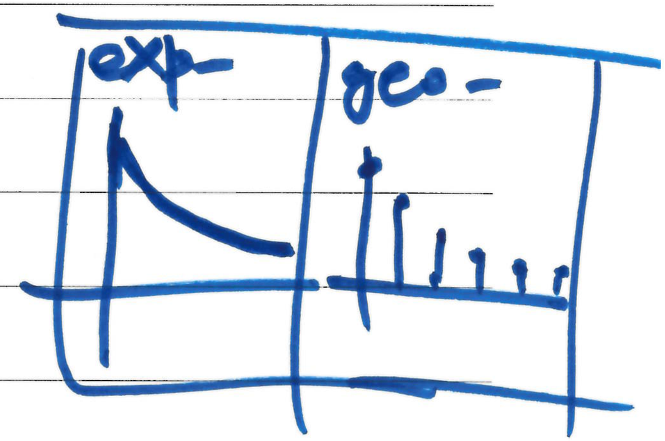
- Consider sampling $x(t)$ at equi-spaced instants in time, every T_s seconds.

$$x[n] = x(t)\Big|_{t=nT_s} = x(nT_s)$$

$$= e^{-anT_s} \cdot u(nT_s) = \left(e^{-aT_s}\right)^n u[n]$$

$$\Rightarrow \underline{x[n] = \alpha^n u[n]}, \quad \alpha = e^{-aT_s}$$

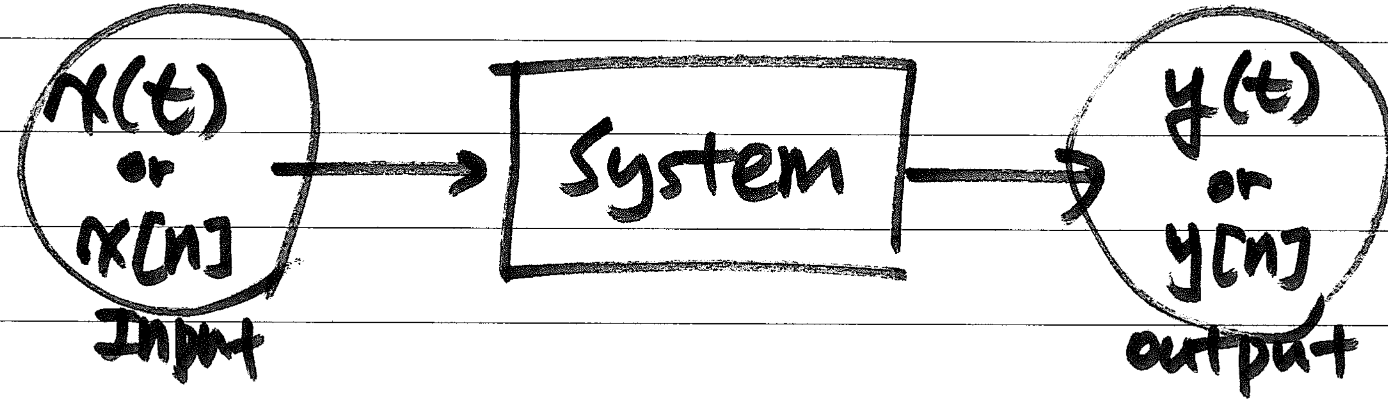




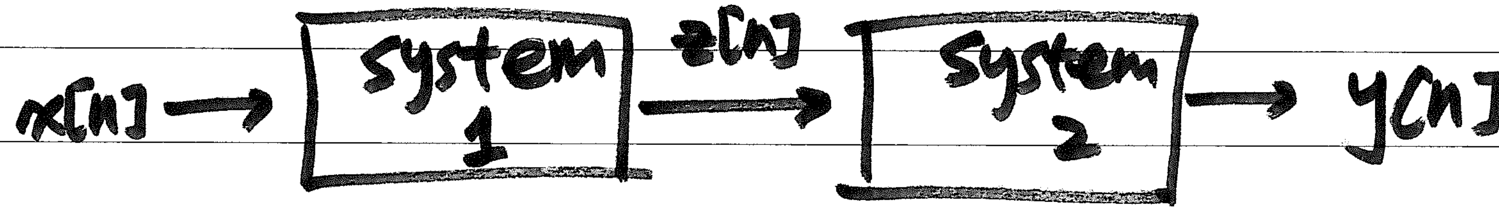
- Sampling a CT exponential signal yields a DT geometric signal (sequence)

1-3. Systems

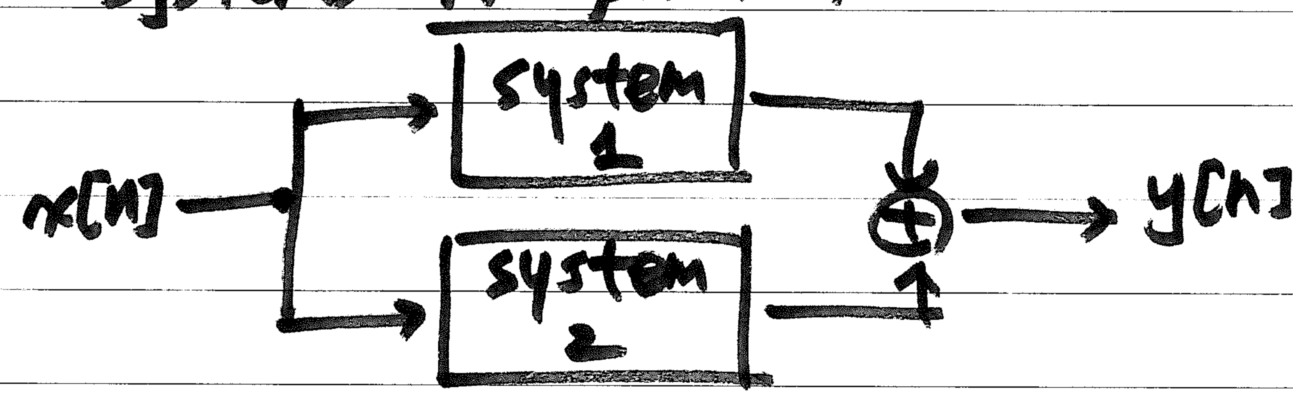
1)



- Systems in series



- Systems in parallel



2) Potential System Properties

- Linear $\quad x_i(t) \rightarrow \boxed{S} \rightarrow y_i(t)$

$i = 1, 2$

$$\underline{a_1 x_1(t) + a_2 x_2(t)} \rightarrow \boxed{S} \rightarrow \underline{a_1 y_1(t) + a_2 y_2(t)}$$

- Time Invariance $\quad x(t) \rightarrow \boxed{S} \rightarrow y(t)$

$$\underline{x(t-t_0)} \rightarrow \boxed{S} \rightarrow \underline{y(t-t_0)}$$

- Causal : $y(t)$ does not depend on future values of $x(t)$

e.g. $y(t) = 2x(t+1)$

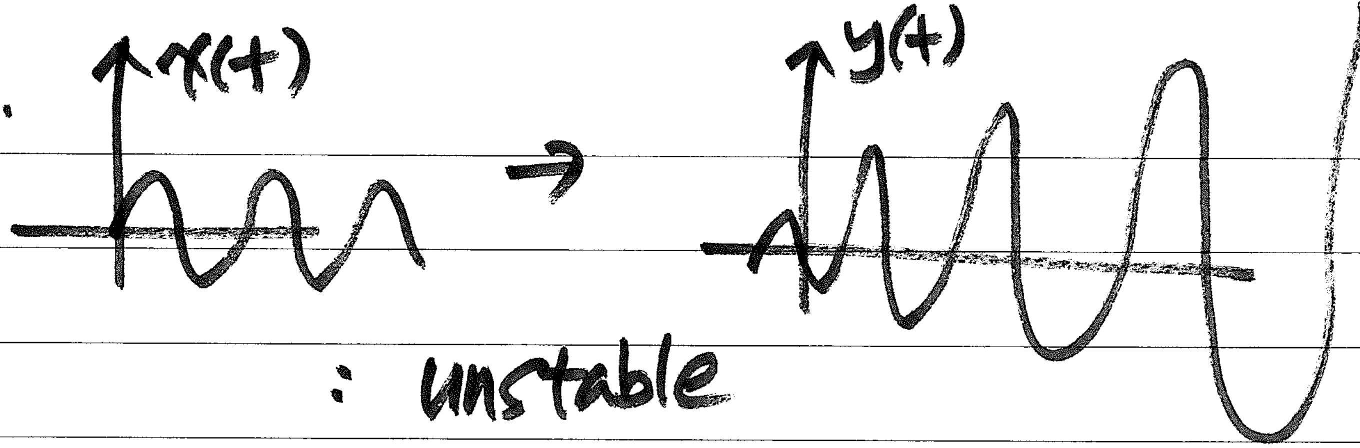
$y(1) = 2x(2)$: not causal

- Stable : $x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$

If $|x(t)| < B_{in}$, then $|y(t)| < B_{out}$, $\forall t$

, where $B_{in} \neq \infty$, $B_{out} \neq \infty$

e.g.



: unstable

- Memoryless : $\underline{y(t_0)}$ only depends on $\underline{x(t_0)}$, $\forall t_0$

- e.g.) $\underline{y(t)} = \frac{1}{2}\left(x(t) + \underline{x(t-1)}\right)$: not memoryless

- Invertible : $x(t) \rightarrow \boxed{S} \rightarrow \boxed{S^{-1}} \rightarrow x(t)$

inverse system

, for any input $x(t)$

3) CT : common systems	Linear	Time-Invariant
$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$	o	o
$y(t) = \int_{t-T_1}^{t+T_2} x(\tau)d\tau$	o	o
$y(t) = x^2(t)$	x	o
$y(t) = x(at)$	o	x
$y(t) = g(t)x(t)$	o	x
$y(t) = \frac{d}{dt}x(t)$	o	o
$y(t) = x(t-t_0)$	o	o
$y(t) = \begin{cases} x(t), & a < x(t) < b \\ b, & x(t) > b \\ a, & x(t) < a \end{cases}$	x	o

* Linear & Time-Invariant : LTI

DT : Common Systems	Linear	Time-Invariant
$y[n] = \sum_{k=-\infty}^{n} x[k]$	O	O
$y[n] = \sum_{k=n-N_1}^{n+N_2} x[k]$	O	O
$y[n] = x^2[n]$	X	O
$y[n] = x[Ln]$	O	X
$y[n] = g[n]\, x[n]$	O	X
$y[n] = x[n] - x[n-1]$	O	O
$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$ (Difference Eq.)	O	O
	(LTI)	

* Eq. : Equation