LAB #1

CSI - Time of Death

Goal: Approximate parameters in a differential equation using calculus; approximate the solution of a differential equation using direction fields; solve the differential equation; compare approximate solutions to true solutions.

Required tools: ***dfield*** ; separable differential equations.

Discussion

You are a forensic expert with the Lafayette PD. You arrive at a crime scene at 12 midnight, finding a body laying on the floor. It is your job to determine the time of death.

You immediately measure the body temperature, finding it to be 76.4°. You also note that the thermostat in the room is set to 70°. Your idea is to extrapolate backwards in time to determine when the body temperature was last 98.6°. You reason that the closer the body temperature gets to room temperature, the slower the body cools. This suggests that the body temperature T satisfies a differential equation of the form

$$\boldsymbol{\frac{dT}{dt} = -k\,(T - 70)} \quad (*)$$

where k is a positive constant which depends on factors such as the size and weight of the victim and t is the number of hours past 12 midnight. To get an idea of how fast the body is cooling, you take a second measurement of the body temperature at 1:00 AM, finding it to be 73.9°.

Assignment

(1) From calculus we know that

$$T'(0) \approx \frac{T(1) - T(0)}{1 - 0}\,.$$

Use this result together with the differential equation $(*)$ to show that $k \approx 0.39$.

(2) Use the "keyboard input" feature of ***dfield*** with initial condition $T(0) = 76.4$ to plot the solution to $(*)$ with $k = 0.39$. Use the "zoom in" feature of ***dfield*** to approximate the number of hours the body has been dead and the time of death to within 2 places after the decimal.

(3) The value of k computed in Part (1) is only approximate. Demonstrate this by using the "zoom in" feature to find the value of $T(1)$ predicted by our model using $k = 0.39$. (It should be different than the observed value of 73.9°.)

(4) Solve the differential equation $(*)$ with the initial condition $T(0) = 76.4°$, with arbitrary k, and then use $T(1) = 73.9°$ to find the precise value of k.

(5) Use ***dfield*** as in Part (2) with $T(0) = 76.4$ to recompute the number of hours the body has been dead and the time of death using the value of k found in Part (4).

(6) Use the solution to the differential equation $(*)$, together with the value of k from Part (4), to find the "exact" number of hours the body has been dead and the "exact" time of death. Compare it with the approximate value found in Part (5).

(7) Your intern disagrees with your answer. The intern points out that from Part (1) the body is losing heat at the rate of $76.4 - 73.9 = 2.5$ degrees per hour. At this rate, the body has been dead

$$\frac{98.6 - 76.4}{2.5} = 8.9 \quad \text{hours},$$

which is more than twice your result in Part (6). What is wrong with the intern's reasoning?

(8) It is discovered that the thermostat in the apartment is programmed to automatically change temperature at 11 PM and that prior to this time, the temperature was set to $\left(65 + \frac{\textbf{seed}}{60}\right)$ where "**seed**" is your personal seed number. Use ***dfield*** to approximate the time of death using this data. You may assume that the change in room temperature from $\left(65 + \frac{\textbf{seed}}{60}\right)$ to 70° occurs instantaneously and that the value of k does not depend on the room temperature.

<u>*Hint*</u> : Using the solution to $(*)$ in Part (4), compute the body temperature at time $t = -1$ and use this as the initial data for a new differential equation.

(9) Note that it is also true that

$$T'(1) \approx \frac{T(1) - T(0)}{1 - 0}.$$

If we take the average of the approximation for $T'(0)$ in Part (1) and the approximation for $T'(1)$ above, we obtain the estimate

$$\frac{T'(0) + T'(1)}{2} \approx \frac{T(1) - T(0)}{1 - 0}.$$

Use this together with the differential equation $(*)$ to find an approximation for k. Is this closer to the true value of k determined in Part (4) ?