ECE301 Signals and Systems
Homework # 3 Solution

$\boxed{2.23}$ $$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

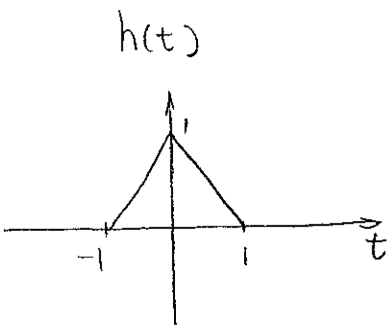
$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= h(t) * x(t) \\ &= h(t) * \sum_{k=-\infty}^{\infty} \delta(t-kT) \\ &= \sum_{k=-\infty}^{\infty} \delta(t-kT) * h(t) \\ &= \sum_{k=-\infty}^{\infty} h(t-kT) \quad (*) \end{aligned}$$

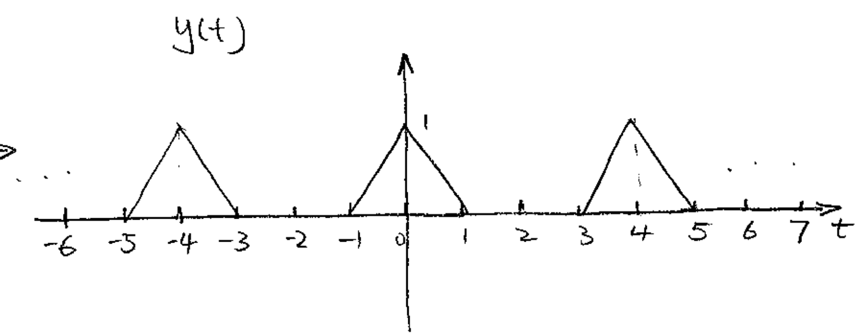
(*) Note that convolution with a delta function simply shifts the original signal to where the delta function is positioned. i.e.

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

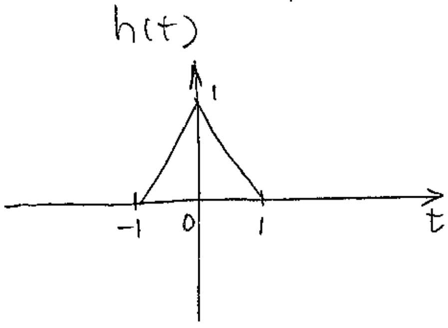
Thus $y(t)$ is a train of shifted $h(t)$ with an interval of T.

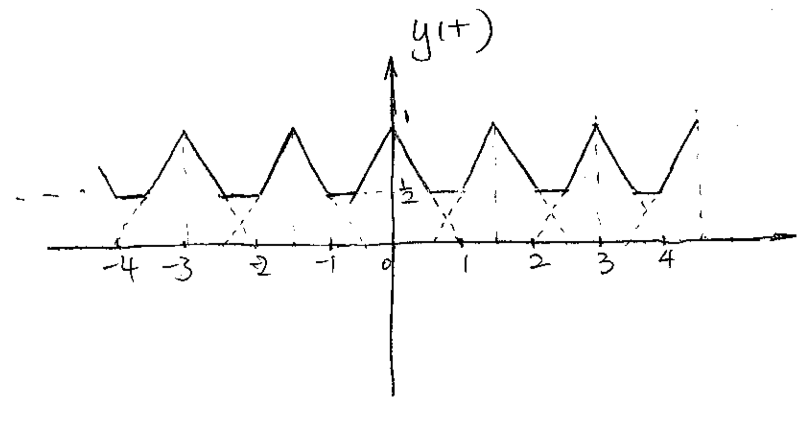
a) $T=4$.





b) $T=\frac{3}{2}$.





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2.24 The overall system response is

$$h[n] = h_1[n] * h_2[n] * h_2[n].$$
$$= h_1[n] * h_3[n].$$

$$h_3[n] = h_2[n] * h_2[n]$$
$$= \delta[n] + 2\delta[n-1] + \delta[n-2].$$

$$\therefore\ h[n] = h_1[n] * h_3[n]$$
$$= h_1[n] + 2h_1[n-1] + h_1[n-2].$$

Plugging in the values in Fig 2.24 (b).

$$h[0] = h_1[0] + 2h_1[-1] + h_1[-2] = 1$$
$$h[1] = h_1[1] + 2h_1[0] + h_1[-1] = 5.$$
$$\vdots$$
$$h[6] = h_1[6] + 2h_1[5] + h_1[4] = 1$$

Since the systems are all causal, $h_1[n] = h_2[n] = 0$, for all $n<0$.

Solving for the equations.

$$h_1[n] = \begin{cases} 0, & n<0, \\ 1, & n=0, \\ 3, & n=1, \\ 3, & n=2 \\ 2, & n=3 \\ 1, & n=4 \\ 0, & n>4 \end{cases}$$

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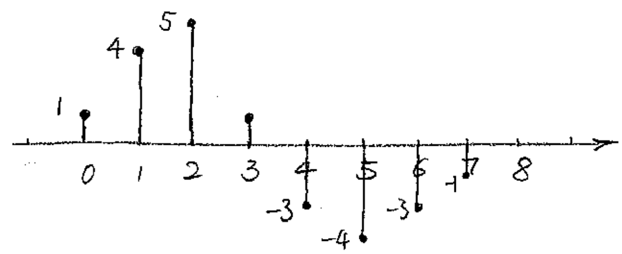
(b). When the input is $x[n] = \delta[n] - \delta[n-1]$, the output is $y[n] = h[n] - h[n-1]$, as depicted in below.



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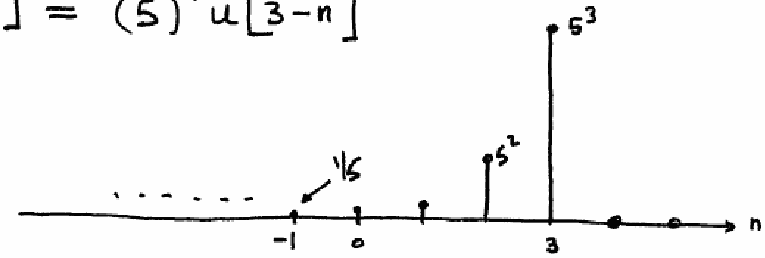
Homework # 3 Solution

$\boxed{\text{O+W 2.28 (d)}}$ Causal and/or stable

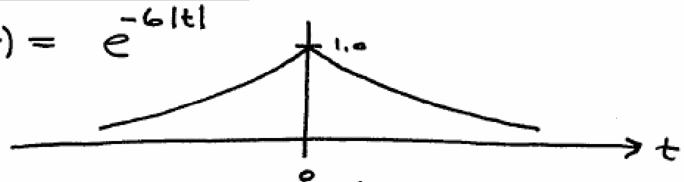
$h[n] = (5)^n u[3-n]$



Clearly noncausal and abs. summable $\Rightarrow$ BIBO stable.

$\boxed{\text{O+W 2.29 (e)}}$ Causal and/or stable.

$h(t) = e^{-6|t|}$



Clearly noncausal and abs. integrable $\Rightarrow$ BIBO stable.

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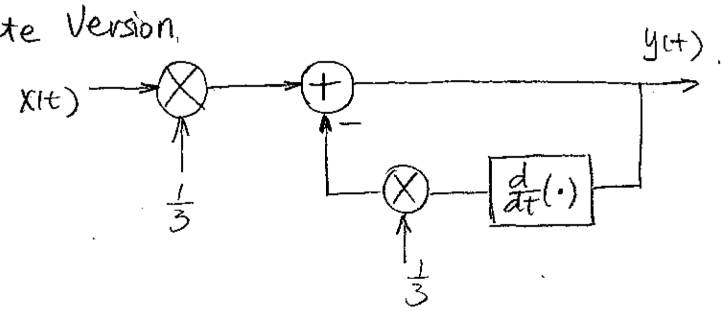
$\boxed{2.39}$ b) $$\frac{dy(t)}{dt} + 3y(t) = x(t)$$

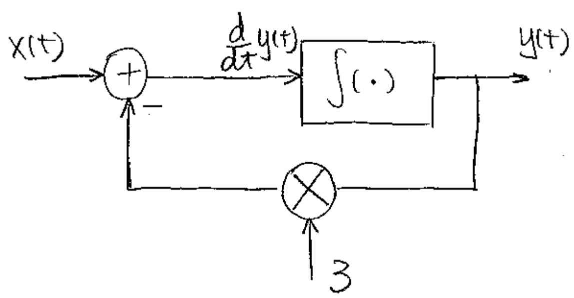
$$y(t) = \frac{1}{3}x(t) - \frac{1}{3}\frac{dy(t)}{dt}.$$

Ditterentiate Version.



Integrator Version.

$$\frac{dy(t)}{dt} = x(t) - 3y(t).$$



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Homework # 3 Solution

$\boxed{\text{O + W 2.48 (a, b, d, f)}}$ True or False

(a) If $h(t)$ is impulse responce of LTI and if $h(t)$ is periodic and nonzero, then system unstable.

True. Must have $\int_{-\infty}^{\infty} |h(t)|\, dt = +\infty$ unless $h(t) \equiv 0$.

(b) The inverse of causal LTI is always causal.

False. Take $h[n] = \delta[n-k]$. Its inverse $h_{inv}[n] = \delta[n+k]$.

(d) DT LTI with $h[n]$ of finite duration is stable.

True. Always have $\sum_{\substack{\text{finite \#} \\ \text{indices}}} |h[n]| < \infty$.

(f) Cascade of noncausal with causal must be overall noncasal.

False. Take $h_1[n] = \delta[n+1]$ and $h_2[n] = \delta[n-1]$

$\Longrightarrow h_1 * h_2[n] = \delta[n]$ which is causal.

$\boxed{\text{O+W } 2.53 \ (c-i, v)}$

(c-i) $y^{(2)}(t) + 3y^{(1)}(t) + 2y(t) = 0 \qquad y(0) = 0, \ y^{(1)}(0) = 2$

Char. eqn. $s^2 + 3s + 2 = (s+1)(s+2)$

$$\therefore \ y(t) = c_1 e^{-t} + c_2 e^{-2t} \qquad t \geq 0.$$

$$0 = c_1 + c_2 \longrightarrow c_1 = -c_2$$

$$2 = -c_1 - 2c_2$$

$$= c_2 - 2c_2 = -c_2 \longrightarrow c_2 = -2, \ c_1 = 2$$

$$y(t) = 2(e^{-t} - e^{-2t})\, u(t).$$

(c-v) $y^{(3)}(t) + y^{(2)}(t) - y^{(1)}(t) - y(t) = 0 \qquad y(0) = 1, \ y^{(1)}(0) = 1$
$y^{(2)}(0) = -2$

Char. eqn. $s^3 + s^2 - s - 1 = 0$

Easy to see $s = 1$ is a root. Therefore has factor $(s-1)$

$$\begin{array}{r|l} & s^2 + 2s + 1 \\ \hline s-1 & s^3 + s^2 - s - 1 \\ & s^3 - s^2 \\ \hline & \quad 2s^2 - s - 1 \\ & \quad 2s^2 - 2s \\ \hline & \qquad s - 1 \end{array}$$

$$\therefore \ (s-1)\underbrace{(s^2 + 2s + 1)}_{(s+1)^2}$$

$$\therefore \quad y(t) = c_1 e^{t} + c_2 e^{-t} + c_3 t e^{-t}$$

$$y^{(1)}(t) = c_1 e^{t} - c_2 e^{-t} + c_3 e^{-t} - c_3 t e^{-t}$$

$$y^{(2)}(t) = c_1 e^{t} + c_2 e^{-t} - c_3 e^{-t} - c_3 e^{-t} + c_3 t e^{-t}$$

$$\therefore \quad y(0) = c_1 + c_2$$

$$y^{(1)}(0) = c_1 - c_2 + c_3$$

$$y^{(2)}(0) = c_1 + c_2 - c_3 - c_3$$

$$1 = c_1 + c_2$$
$$1 = c_1 - c_2 + c_3$$
$$-2 = c_1 + c_2 - 2c_3$$

subtract 3rd from 1st: $3 = 2c_3 \rightarrow c_3 = 3/2$

add 1st + 2nd: $2 = 2c_1 + 3/2 \rightarrow 2c_1 = \frac{1}{2} \rightarrow c_1 = 1/4$

from 1st: $c_2 = 1 - c_1 = 3/4$.

$$\therefore \; y(t) = \left(\frac{1}{4}e^{t} + \frac{3}{4}e^{-t} + \frac{3}{2}te^{-t}\right)u(t).$$

$\boxed{\text{O+W } 2.54\,(c-i,\ iv)}$

(c-i) $y[n] + \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 0 \qquad y[0] = 1,\; y[-1] = -6$

Char. eqn. $r^2 + \frac{3}{4}r + \frac{1}{8} = 0$

$$r = \frac{-3/4 \pm \sqrt{\frac{9}{16} - \frac{8}{16}}}{2} = -\frac{3}{8} \pm \frac{1}{8} = -\frac{1}{2}, -\frac{1}{4}$$

$$y[n] = A\left(-\tfrac{1}{2}\right)^n + B\left(-\tfrac{1}{4}\right)^n$$

$$1 = A + B \implies B = 1 - A$$
$$-6 = -2A - 4B$$

$$-6 = -2A - 4(1-A) = 2A - 4 \longrightarrow -2 = 2A$$
$$A = -1$$
$$B = 2$$

$$y[n] = \left(-\left(-\tfrac{1}{2}\right)^n + 2\left(-\tfrac{1}{4}\right)^n\right)u[n].$$

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(c-iv) $\quad y[n] - \frac{\sqrt{2}}{2} y[n-1] + \frac{1}{4} y[n-2] = 0 \qquad y[0] = 0,\ y[-1] = 1$

char. eqn.

$$r^2 - \frac{\sqrt{2}}{2} r + \frac{1}{4} = 0$$

$$r = \frac{\frac{\sqrt{2}}{2} \pm \sqrt{\frac{2}{4} - 1}}{2} = \frac{\sqrt{2}}{4} \pm j\,\frac{1}{2\sqrt{2}}$$

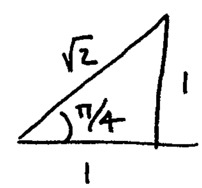
$$= \frac{\sqrt{2}}{4}(1 \pm j)$$

$$= \frac{1}{2} e^{\pm j \pi/4}$$

$$\therefore \quad y[n] = A\left(\tfrac{1}{2}\right)^n \cos(n\pi/4) + B\left(\tfrac{1}{2}\right)^n \sin(n\pi/4)$$

$$0 = A$$

$$1 = 2\cancel{A}^{\,0} \cos(-\pi/4) + 2B \sin(-\pi/4)$$



$$1 = 2B\left(-\frac{1}{\sqrt{2}}\right)$$

$$B = -\frac{\sqrt{2}}{2}$$

$$\therefore \quad y[n] = -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)^n \sin\left(\frac{n\pi}{4}\right) u[n].$$