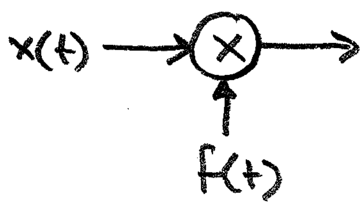
Problem 1

(a) In all cases the overall system is linear since mult. by a fixed signal is linear



$\longrightarrow z(t) = x(t)\, f(t)$

clearly linear in the input Signal $x(t)$

The above is then cascaded with LTI, which is linear. The cascade of linear systems is always linear.

None of the cases are time invariant because of the presence of the time-varying mixer followed only by a LTI system (which cannot compensate the time variation).

A mixer is also memoryless, causal and BIBO. Therefore the overall system will have these properties if the following LTI system does:

(i) $h(t) = \delta(t)$ is memoryless, causal, BIBO

(ii) $h(t) = u(t)$ has memory, is causal, is <u>not</u> BIBO.

(iii) $h(t) = e^{-2t}u(t)$ has memory, is causal, is BIBO.

(iv) $h(t) = e^{-2|t|}$ has memory, is <u>not</u> causal, is BIBO.

(b) Can do the two parts at the same time

$$y(t) = e^{-j1000t} \int x(\tau) e^{j1000\tau} h(t-\tau)\, d\tau$$

$$= e^{-j1000t} \int x(t-\tau) e^{j1000(t-\tau)} h(\tau)\, d\tau$$

$$y(t) = \underbrace{e^{-j1000t}\, e^{j1000t}}_{1} \underbrace{\int x(t-\tau)\, e^{-j1000\tau} h(\tau)\, d\tau}$$

Is a convolution of
$x(t)$ and

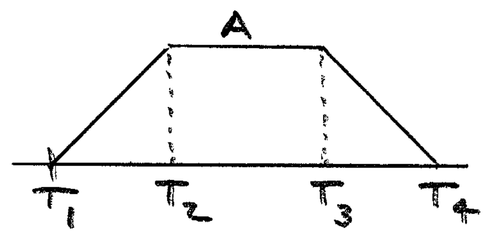
$$h_{overall}(t) = e^{-j1000t}\, h(t)$$

$\therefore$ LTI with above impulse response.

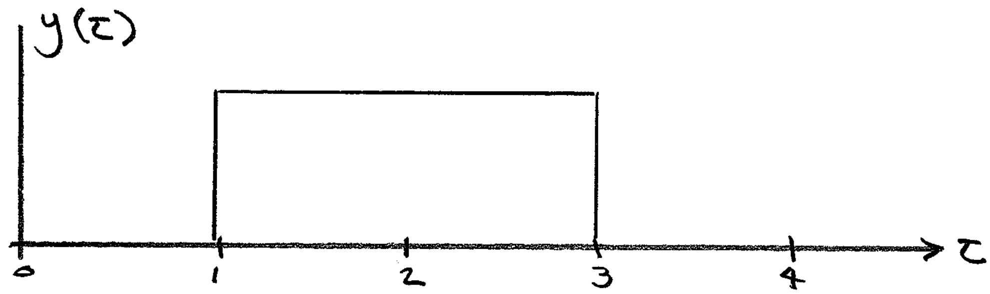
Problem 2 $\quad z(t) = x * y(t)$

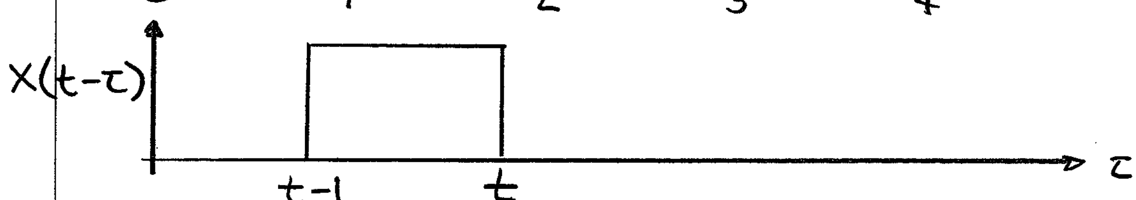
$$= \int x(t-\tau)\,y(\tau)\,d\tau$$

Draw pictures to determine limits and break points but common sense (and some experience) tells me the plot of $z(t)$ has a shape like:



Thus only need figure out—
amplitude A
and times
$T_1 \; T_2 \; T_3 \; T_4$





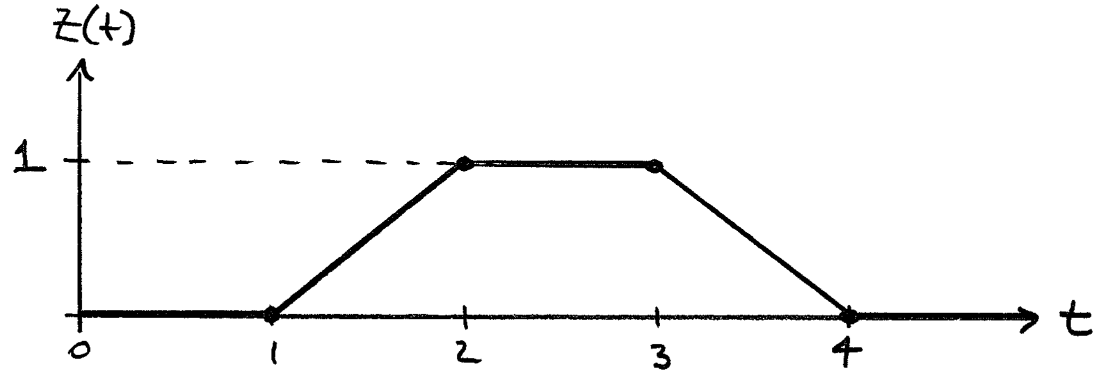
Case $\quad t \le 1 \longrightarrow$ no overlap $\longrightarrow z(t) = 0$

$1 \le t \le 2 \longrightarrow$ overlap with linearly growing area $\longrightarrow z(t) \sim t$

$2 \le t \le 3 \longrightarrow$ overlap is constant $\longrightarrow z(t) =$ area under $x(t-\tau)$
$= 1.$

$3 \le t \le 4 \rightarrow$ overlap with linearly decreasing area $\rightarrow z(t) \sim -t$

$t \ge 4 \;\longrightarrow$ no overlap $\longrightarrow z(t) = 0.$



From picture can get equation for $z(t)$:

$$z(t) = \begin{cases} 0 & \text{for} \quad t \leq 1 \\ t-1 & \text{"} \quad 1 \leq t \leq 2 \\ 1 & \text{"} \quad 2 \leq t \leq 3 \\ -t+4 & \text{"} \quad 3 \leq t \leq 4 \\ 0 & \text{"} \quad t \geq 4 \end{cases}$$

Problem 3

(a) Let $z[n]$ = input of unit delay. Then

$$z[n] = x[n] + \frac{1}{2} z[n-1]$$

which is the desired difference equation.

(b) To find the impulse response $h[n]$ we assume the system is at rest at $n=-1$ ie

$$z[-1] = 0$$

and let $x[n] = \delta[n]$. Then the difference equation becomes

$$h[n] - \frac{1}{2} h[n-1] = \delta[n] \qquad h[-1] = 0$$

Iterating once

$$h[0] - \frac{1}{2} h[-1] = 1 \longrightarrow h[0] = 1$$

Then for $n > 0$

$$h[n] - \frac{1}{2} h[n-1] = 0 \qquad \text{with } h[0] = 1$$

Char. eqn is:

$$1 - \frac{1}{2} r^{-1} = 0 \implies r - \frac{1}{2} = 0 \implies \text{root} = \frac{1}{2}$$

$$\therefore h[n] = c\left(\frac{1}{2}\right)^n \qquad n \geq 0$$

$$h[0] = 1 \implies c = 1$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

(c) $y[n] = \frac{1}{2} z[n] + z[n-1]$

$$\therefore \; h_{overall}[n] = \frac{1}{2} h[n] + h[n-1]$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$$h_{overall}[0] = \frac{1}{2}$$

$$h_{overall}[1] = \frac{1}{4} + 1 = \frac{5}{4}$$

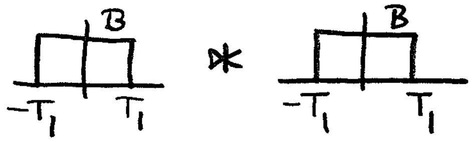
$$h_{overall}[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} \quad n \geq 1$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1}{2} + 2\right]$$

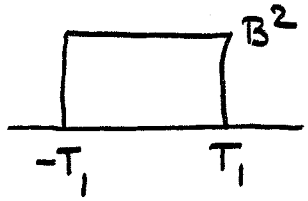
$$= \frac{5}{2}\left(\frac{1}{2}\right)^n \quad n \geq 1$$

Problem 4

(a) An easy way to do this is to recognize that

$x(t) =$  (convolution of 2 identical rectangular pulses)

This convolution will be nonzero for $|t| \leq 2T_1$, its peak will occur when the pulses are just overlapping completely ie @ $t=0$ and the amplitude at $t=0$ will be the area under

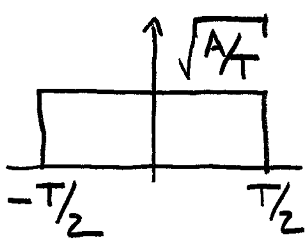


$$\therefore \quad x(0) = A = 2T_1B^2$$

$$2T_1 = T$$

$$\therefore \quad A = TB^2 \longrightarrow B = \sqrt{A/T}$$

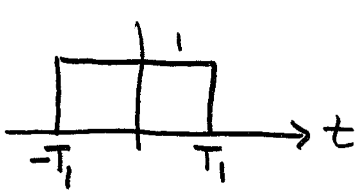
Hence $x(t) = x_1(t) * x_1(t)$ where



$\longleftarrow x_1(t)$.

From convolution property $X(j\omega) = X_1^2(j\omega)$

To get $X_1(j\omega)$ use the table entry



$$\longleftrightarrow \quad \frac{2\sin \omega T_1}{\omega}$$

Setting $T_1 = T/2$ and scaling amplitude have

$$X_1(j\omega) = \sqrt{\frac{A}{T}}\ \frac{2\sin(\omega T/2)}{\omega}$$

$$= \sqrt{\frac{A}{T}}\ T\ \frac{\sin(\omega T/2)}{(\omega T/2)}$$

$$= \sqrt{AT}\ \frac{\sin(\omega T/2)}{(\omega T/2)} \longrightarrow$$

has peak $= \sqrt{AT}$ @ $\omega = 0$ and zeros @ $\omega = \frac{2\pi}{T}k \quad k = \pm 1, \pm 2 \ldots$

$$\therefore\ X(j\omega) = X_1^2(j\omega)$$

$$= AT\ \frac{\sin^2(\omega T/2)}{(\omega T/2)^2}$$

(b)