

Midterm Examination 1
ECE 301
Division 3, Fall 2007
Instructor: Mimi Boutin

Instructions:

1. Wait for the “BEGIN” signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete the 5 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately.** Anyone caught writing after the exam is over will get a grade of zero.
3. This booklet contains 9 pages. The last three pages contain a table of formulas and properties. You may tear out these three pages **once the exam begins.**
4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden. Ipods and PDAs are not allowed either.

Name: _____

Email: _____

Signature: _____

Itemized Scores

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No
If $y(t) = x(2t)$, is the system causal?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = (t + 2)x(t)$, is the system causal?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = x(-t^2)$, is the system causal?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = x(t) + t - 1$, is the system memoryless?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = x(t^2)$, is the system memoryless?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = x(t/3)$, is the system stable?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = tx(t/3)$, is the system stable?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = \int_{-\infty}^t x(\tau)d\tau$, is the system stable?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = \sin(x(t))$, is the system time invariant?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = u(t) * x(t)$, is the system LTI?	<input type="checkbox"/>	<input type="checkbox"/>
If $y(t) = (tu(t)) * x(t)$, is the system linear?	<input type="checkbox"/>	<input type="checkbox"/>

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t + 2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all \sum signs disappear.)

(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

(15 pts) 4. Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \leq t \leq 2 \\ 0, & 2 < t \leq 4 \end{cases} .$$

(Simplify your answer as much as possible.)

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

input	output
$x_0[n] = \delta[n]$	$\rightarrow y_0[n] = \delta[n - 1],$
$x_1[n] = \delta[n - 1]$	$\rightarrow y_1[n] = 4\delta[n - 2],$
$x_2[n] = \delta[n - 2]$	$\rightarrow y_2[n] = 9\delta[n - 3],$
$x_3[n] = \delta[n - 3]$	$\rightarrow y_3[n] = 16\delta[n - 4],$
	\vdots
$x_k[n] = \delta[n - k]$	$\rightarrow y_k[n] = (k + 1)^2\delta[n - (k + 1)]$ for any integer k .

(10 pts) **a)** Can this system be time-invariant? Explain.

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n - 1]$?

Facts and Formulas

1 CT Signal Energy and Power

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (1)$$

$$P_\infty = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad (2)$$

2 Fourier Series of CT Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(\frac{2\pi}{T})t} \quad (3)$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk(\frac{2\pi}{T})t} dt \quad (4)$$

3 Properties of CT Fourier Series

Let $x(t)$ be a periodic signal with fundamental period T and fundamental frequency ω_0 . Let $y(t)$ be another periodic signal with the same fundamental period T and fundamental frequency ω_0 . Denote by a_k and b_k the Fourier series coefficients of $x(t)$ and $y(t)$ respectively.

	<i>Signal</i>	<i>FT</i>
Linearity:	$\alpha x(t) + \beta y(t)$	$\alpha a_k + \beta b_k$ (5)
Time Shifting:	$x(t - t_0)$	$e^{-jk\omega_0 t_0} a_k$ (6)
Conjugation:	$x^*(t)$	a_{-k}^* (7)
	$x(t)$ real and even	a_k real and even (8)
	$x(t)$ real and odd	a_k pure imaginary and odd (9)

Parseval's Relation: $\frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$ (10)

4 DT Signal Energy and Power

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \quad (11)$$

$$P_\infty = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2 \quad (12)$$

5 Fourier Series of DT Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \quad (13)$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk(\frac{2\pi}{N})n} \quad (14)$$

6 Properties of DT Fourier Series

Let $x[n]$ be a periodic signal with fundamental period N and fundamental frequency ω_0 . Let $y[n]$ be another periodic signal with the same fundamental period N and fundamental frequency ω_0 . Denote by a_k and b_k the Fourier series coefficients of $x(t)$ and $y(t)$ respectively.

	<i>Signal</i>	<i>FT</i>
Linearity:	$\alpha x[n] + \beta y[n]$	$\alpha a_k + \beta b_k$ (15)
Time Shifting:	$x[n - n_0]$	$e^{-jk\omega_0 n_0} a_k$ (16)
Conjugation:	$x^*[n]$	a_{-k}^* (17)
	$x[n]$ real and even	a_k real and even (18)
	$x[n]$ real and odd	a_k pure imaginary and odd (19)

Parseval's Relation: $\frac{1}{N} \sum_{n=0}^{N-1} |x(t)|^2 dt = \sum_{k=0}^{N-1} |a_k|^2$ (20)

7 Properties of LTI systems

- LTI systems commute.
- The response of an LTI system with unit impulse response h to a signal x is the same as the response of an LTI system with unit impulse response x to the signal h .
- An LTI system consisting of a cascade of k LTI systems with unit impulse responses h_1, h_2, \dots, h_k respectively, is the same as an LTI system with unit impulse response $h_1 * h_2 * \dots * h_k$.
- The response of a CT LTI system with unit impulse response $h(t)$ to the signal e^{st} is $H(s)e^{st}$ where $H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau} d\tau$.
- The response of a DT LTI system with unit impulse response $h[n]$ to the signal z^n is $H(z)z^n$ where $H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$.