

3.7 Properties of DT Fourier Series

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- * Linearity
- * Timeshift
- * Parseval's relation

Examples of Questions

A signal $x(t)$ is such that - find the signal $x(t)$

1. $x(t)$ is real

2. $x(t)$ is periodic with period $T=4$

3. $a_k = 0$ for $|k| > 1$

4. The signal with Fourier Coefficients $b_k = a_k e^{-jk\frac{\pi}{2}}$ is odd

5. $\frac{1}{4} \int_0^4 |x(t)|^2 dt = \frac{1}{2}$

solution

From 2 and 3 we know

$$\begin{aligned}x(t) &= a_0 + a_1 e^{j\frac{2\pi}{4}t} + a_{-1} e^{-j\frac{2\pi}{4}t} \\ &= a_0 + a_1 e^{j\frac{\pi}{2}t} + a_{-1} e^{-j\frac{\pi}{2}t}\end{aligned}$$

real signal $\Rightarrow a_k = a_{-k}^*$

$$\begin{aligned}so \quad a_0 &= a_{-0}^* = a_0^* \quad \text{conjugated} \\ a_1 &= a_{-1}^* \\ a_1^* &= a_{-1}\end{aligned}$$

$$x(t) = a_0 + a_1 e^{j\frac{\pi}{2}t} + a_1^* e^{-j\frac{\pi}{2}t} = a_0 + 2 \operatorname{Re} (a_1 e^{j\frac{\pi}{2}t})$$

To use 4, use 3.5.3 and 3.5.2

$x(-t)$ has Fourier coeff a_{-k}
 $x(t-t_0)$ has Fourier coeff $a_k e^{jk\frac{\omega}{2}t_0}$

Let $y(t) = x(-t)$
Let $z(t) = y(t-t)$

F.C. of $y(t) = C_k = a_{-k}$
F.C. of $z(t) = b_k = C_k e^{-jk\frac{\omega}{2}t} = a_{-k} e^{-jk\frac{\omega}{2}t}$

∴ the signal with coeff b_k is obtained with transforming $x(t)$ into $x(at + t_0)$