The **golden ratio** is a ratio such that, given two quantities a and b,

$$\frac{a+b}{a} = \frac{a}{b}$$

We can solve this equation to find an explicit quantity for the ratio.

$$LHS = \frac{a}{a} + \frac{b}{a} = 1 + \frac{b}{a}$$
$$1 + \frac{b}{a} = \frac{a}{b}$$

We set the ratio equal to a certain quantity given by r.

$$r \equiv \frac{a}{b}$$

Then we can solve for the ratio numerically.

$$1 + \frac{1}{r} = r$$
$$r + 1 = r^2$$

We can see from the above result that the golden ratio can also be described as a ratio such that in order to get the square of the ratio, you add one to the ratio.

$$r^2 - r - 1 = 0$$

We can then apply the quadratic formula to solve for the roots of the equation.

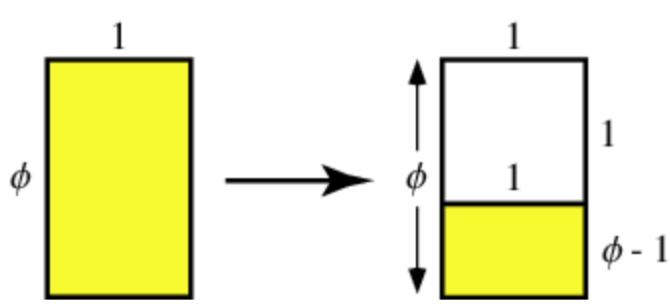
$$r = \frac{1 \pm \sqrt{1^2 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

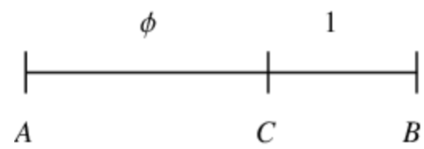
The positive root is then the golden ratio.

$$\frac{1+\sqrt{5}}{2} = 1.618 \ldots \equiv \phi$$

The **golden ratio**, ϕ, is sometimes also called the **golden mean** or the **golden section**. The golden ratio can be frequently observed in man-made objects, though they are generally "**imperfectly golden**" – that is, the ratio is approximately the golden ratio, but not exactly. Some everyday examples include: credit cards, $w/h = 1.604$, and laptop screens, $w/h = 1.602$ (Tannenbaum 392).

Visualizations of the golden ratio can be seen below (Weisstein):





MathIsFun also has an interactive display that can construct a rectangle in the golden ratio given a certain fixed width or length.