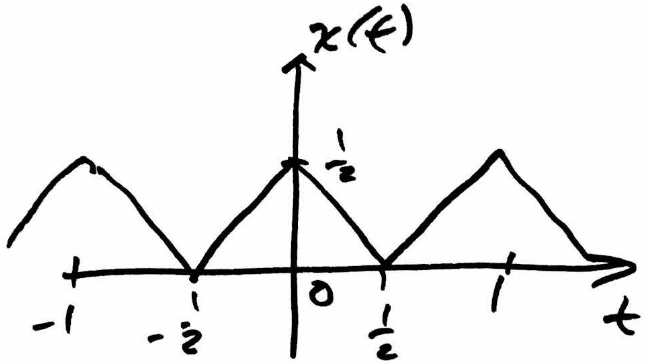
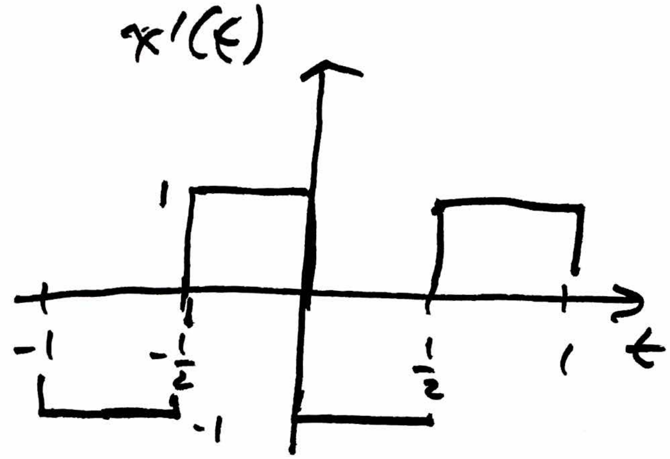
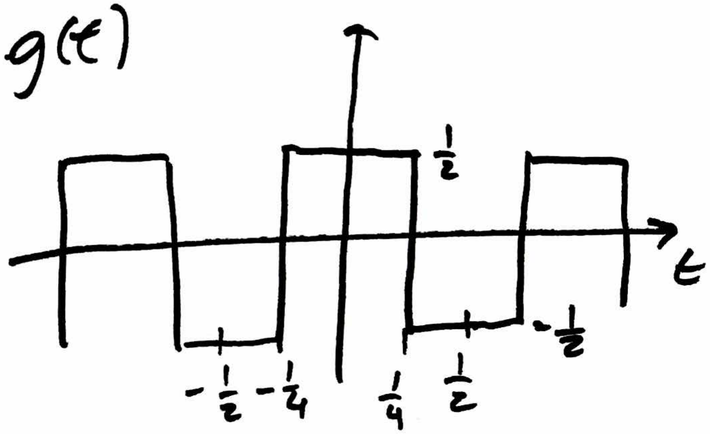
Ex







$$g(t) \overset{FS}{\longleftrightarrow} b_k = \frac{1}{\pi k}\sin\left(\frac{\pi}{2}k\right) \quad k \neq 0$$

$$b_0 = 0$$

$$x'(t) = 2g\left(t+\frac{1}{4}\right)$$

FS properties:

Time shift $\quad x(t-t_0) \overset{FS}{\longleftrightarrow} a_k e^{-jk\left(\frac{2\pi}{T}\right)t_0}$

Linearity (Homogeneity) $\quad Ax(t) \overset{FS}{\longleftrightarrow} A a_k$

$$x'(t) \overset{FS}{\longleftrightarrow} c_k = 2 \cdot b_k e^{-jk2\pi\left(-\frac{1}{4}\right)}$$

$$= \frac{2}{\pi k}\sin\left(\frac{\pi}{2}k\right)e^{jk\frac{\pi}{2}}$$

$$a_0 = \frac{1}{4}$$

$$x(t) \overset{FS}{\longleftrightarrow} \frac{1}{j2\pi k} a_k = \frac{1}{j\pi^2 k^2}\sin\left(\frac{\pi}{2}k\right)e^{jk\frac{\pi}{2}} \quad (k \neq 0)$$

Fourier Series representation for periodic DT signals

$$x[n] = x[n+N]$$

$$x[n] \xleftrightarrow{FS} a_k$$

Analysis: $$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{T_0}n}$$

↑ over any period

Synthesis: $$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk\frac{2\pi}{T_0}n}$$

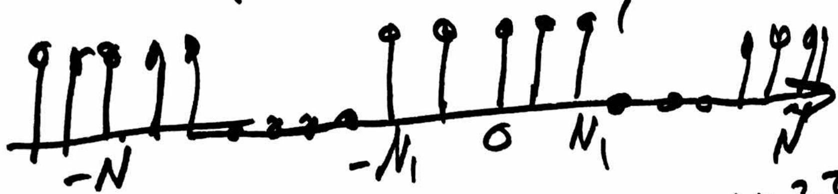
This is a finite series for DT signals.

★ There are no issues with convergence.

There are properties analagous to the CT case listed on page 221 (linearity, time shift, ...).

★ $a_k = a_{k-N} \Rightarrow$ The FS coefficients are periodic.

Example 3.12



periodic with period N

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\frac{2\pi}{N}n} = \frac{1}{N}\sum_{n=-N_1}^{N_1} e^{-jk\frac{2\pi}{N}n}$$

$n' = n + N_1$

$n = n' - N_1$

$$= \frac{1}{N}\sum_{n'=0}^{2N_1} e^{-jk\frac{2\pi}{N}(n'-N_1)} = \frac{1}{N} e^{jk\frac{2\pi}{N}N_1}\, \frac{1-e^{-jk\frac{2\pi}{N}(2N_1+1)}}{1-e^{-jk\frac{2\pi}{N}}}$$

$$\boxed{\begin{aligned} 1-e^{-j\theta} &= e^{-j\frac{\theta}{2}}\left(e^{j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}}\right)\cdot\frac{2j}{2j} \\ &= e^{-j\frac{\theta}{2}}\sin\left(\frac{\theta}{2}\right)\cdot 2j \end{aligned}}$$

$$= \frac{1}{N} e^{jk\frac{2\pi}{N}N_1}\, \frac{e^{-jk\frac{\pi}{N}(2N_1+1)}}{e^{-jk\frac{\pi}{N}}}\, \frac{e^{jk\frac{\pi}{N}(2N_1+1)} - e^{-jk\frac{\pi}{N}(2N_1+1)}}{e^{jk\frac{\pi}{N}} - e^{-jk\frac{\pi}{N}}}$$

$$= \frac{1}{N}\, \underbrace{e^{jk\frac{\pi}{N}(2N_1-2N_1-1+1)}}_{e^{j0}}\, \frac{\sin\left(k\frac{\pi}{N}(2N_1+1)\right)}{\sin\left(k\frac{\pi}{N}\right)}$$

$$= \frac{1}{N}\, \frac{\sin\left(k\frac{\pi}{N}(2N_1+1)\right)}{\sin\left(k\frac{\pi}{N}\right)}$$

Ex 3.14

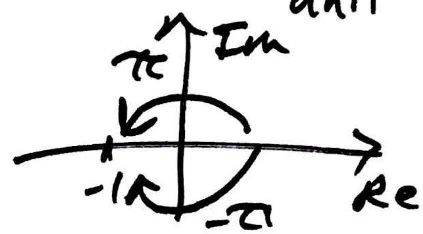
1. $x[n]$ is periodic with period 6
2. $\sum_{n=0}^{5} x[n] = 2$
3. $\sum_{n=2}^{7} (-1)^n x[n] = 1$
4. $x[n]$ has the minimum power necessary to fulfill the requirements.

Find $a_k, x[n]$

1) $\rightarrow$ $x[n] = \sum_{k=\langle 6\rangle} a_k e^{jk\frac{2\pi}{6}n}$ $\quad a_k = \frac{1}{6}\sum_{n=\langle 6\rangle} x[n]e^{-jk\frac{\pi}{3}n}$

2) $a_0 = \frac{1}{6}\sum_{n=0}^{5} x[n]e^{-j0} = \frac{1}{6}\cdot 2 = \frac{1}{3}$

unit circle



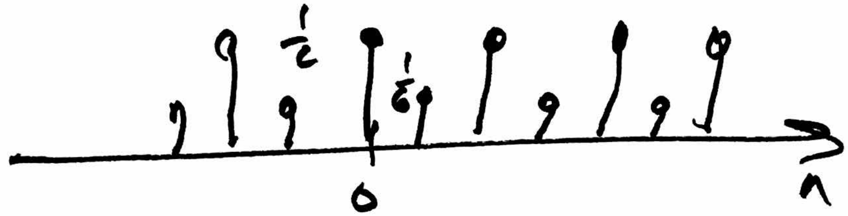
3) $(-1)^n = (e^{j\pi})^n = e^{j\pi n} = e^{-j\pi n}$

$a_k\Big|_{k=3} = \frac{1}{6}\sum_{n=\langle 6\rangle} x[n]e^{-j\pi n} = \frac{1}{6}\cdot 1 = \frac{1}{6} = a_3$

4) $P = \sum_{k=\langle N\rangle} |a_k|^2 \quad a_1 = a_2 = a_4 = a_5 = 0$

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\frac{\pi}{3}n}$$

$$= \frac{1}{3}e^{j0} + \frac{1}{6}e^{j\pi n} = \frac{1}{3} + \frac{1}{6}(-1)^n$$



Ex 3.10

$$x[n] = \sin \omega_0 n \qquad \omega_0 = \frac{2\pi}{N} \text{ for some integer } N$$

$$= \frac{1}{2j} e^{j\frac{2\pi}{N}n} - \frac{1}{2j} e^{-j\frac{2\pi}{N}n}$$

Find the FS coefficients:

Can directly the analysis equation, but sometimes easier when you have $e^{j\theta n}$ to use synthesis.

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\frac{2\pi}{N}n} = \frac{1}{2j} e^{j\frac{2\pi}{N}n} - \frac{1}{2j} e^{-j\frac{2\pi}{N}n}$$

Match terms

$$a_1 = \frac{1}{2j} \qquad a_{-1} = -\frac{1}{2j}$$