ECE 302 Homework 5
Due July 19, 2016

Reading assignment: chapter 5, sections 5.1 - 5.8.

1. The amplitudes of signals X and Y have joint pdf:

$$f_{X,Y}(x,y) = k(x+2y), \quad 0 < x < 1,\, 0 < y < 1,$$

where k is a constant.

   (a) Find k.
   (b) Find the marginal pdfs.
   (c) Find $\Pr(X < Y)$.
   (d) Find $\Pr(X + Y \leq 1)$.
   (e) Find the mean values of X and Y.
   (f) Find the correlation and covariance of X and Y.
   (g) Determine whether X and Y are independent, orthogonal, or uncorrelated.

2. Repeat Problem 1 with random variables X and Y with joint pdf:

$$f_{X,Y}(x,y) = kx(1-x)y, \quad 0 < x < 1,\, 0 < y < 1,$$

where k is a constant.

3. Consider the random variables X and Y with joint pdf as in Problem 1.

   (a) Find $f_{Y|X}(y|x)$.
   (b) Find $\Pr(Y > X|X = x)$.
   (c) Find $\Pr(Y > X)$ using part b.
   (d) Find $\mathbb{E}[Y|X = x]$.

4. Let X and Y be random variables with joint pdf:

$$f_{X,Y}(x,y) = ke^{-(x+y)}, \quad 0 < y < x < \infty,$$

where k is a constant.

(a) Are X and Y independent? Explain why or why not.

(b) Find the value of k.

(c) Let $U = X + Y$. Find the pdf of U.

(d) Let $V = Y/X$. Find the pdf of V.

5. A student throws a dart at a dartboard. Let X and Y be independent random variables denoting the x-coordinate and y-coordinate, respectively, of the dart measured from the center of the dartboard. Assume that X and Y are both Gaussian random variables with mean 0 and variance σ^2.

(a) Find the probability that the distance from the center of the dartboard to the dart is greater than r, where $r \geq 0$. Hint: Use polar coordinates.

(b) Let R be the random variable denoting the distance from the center of the dartboard to the dart. Find the pdf of R. What is the name of the distribution of this random variable?