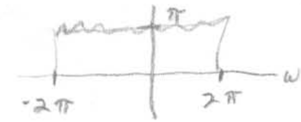
1. Determine whether each of the following signals is band limited. (Answer yes/no and justify.) If they are band limited, specify their Nyquist rate.

(5 pts) a) $x_1(t) = \frac{\sin(2\pi t)}{t} \longrightarrow \pi\left(\frac{\sin(2\pi t)}{\pi t}\right) \xrightarrow{\mathcal{F}} \pi u(\omega+2\pi) - \pi u(\omega-2\pi)$



$NQ = 2\omega_{max}, \ \omega_{max} = 2\pi \Rightarrow \boxed{NQ = 4\pi}$

(5 pts) b) $x_2(t) = 3\,(u(t+2) - u(t-2)) \xrightarrow{\mathcal{F}} 3\left(\frac{2\sin(\omega 2)}{\omega}\right)$

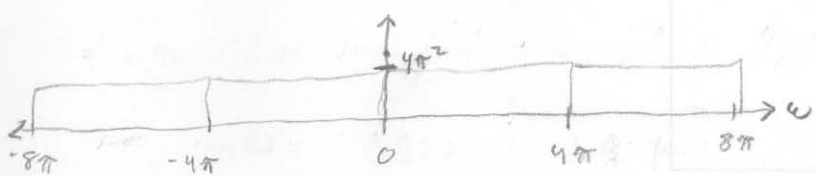
There is no value ω_m for which $\mathcal{F}(x_2(t)) = 0$ when $\omega > \omega_m$

$\therefore$ $x_2(t)$ is NOT BAND LIMITED

(5 pts) c) $x_3(t) = \frac{\sin(2\pi t)}{t} \sum_{n=-\infty}^{\infty} \delta(t - \frac{n}{2})$

$\xrightarrow{\mathcal{F}}$ $= \pi\left(u(\omega+2\pi) - u(\omega-2\pi)\right) * \frac{2\pi}{1/2} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{1/2}\right)$, by (14)(19)(22)

$= 4\pi^2\left((u(\omega+2\pi) - u(\omega-2\pi)) * \sum_{k=-\infty}^{\infty} \delta(\omega - 4\pi k)\right)$



$\therefore$ ω_m does not exist. NOT BAND LIMITED

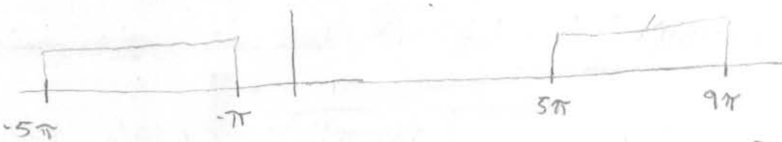
(5 pts) d) $x_4(t) = \left(\frac{\sin(2\pi t)}{t}\right)^2 \xrightarrow{\mathcal{F}} \pi\left(u(\omega+2\pi) - u(\omega-2\pi)\right)$, but since it is squared,

the frequency is doubled and $\omega_{max} = 2 \cdot 2\pi = 4\pi$

$NQ = 2 \cdot \omega_m = 2 \cdot 4\pi = \boxed{8\pi = NQ}$

(5 pts) d) $x_5(t) = \frac{\sin(2\pi t)}{t}\cos(5\pi t) \to \left(\frac{e^{j5\pi t} + e^{-j5\pi t}}{2}\right)$

$\xrightarrow{\mathcal{F}} \pi\left(u(\cancel{w}\omega+2\pi) - u(\omega-2\pi)\right) * \frac{1}{2}\left(2\pi\delta(\omega-5\pi) + 2\pi\delta(\omega+5\pi)\right)$, by (13) (10) (14) (17) (19)



$\therefore |X_5(\omega)| = 0$ when $|\omega| > 9\pi$ $\quad NQ = 2\omega_m \quad \omega_m = 9\pi \quad \boxed{NQ = 18\pi}$

(5 pts) e) $x_6(t) = \frac{\sin(2\pi t)}{t} e^{j5\pi t}$

$\xrightarrow{\mathcal{F}} \pi\left(u(t+2\pi) - u(t-2\pi)\right) * 2\pi\delta(\omega-5\pi)$, by (14)(17)(19)

$= 2\pi^2\left(u(t+2\pi) - u(t-2\pi)\right) * \delta(\omega-5\pi)$

$\omega_m = 9\pi$ $\quad \boxed{NQ = 18\pi}$

(15 pts) **2.** Using the definition of the z-transform (i.e. do not simply take the answer from the table), obtain the z-transform (with its ROC) of

$$x[n] = 3^n u[-n-1]$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} 3^n u[-n-1] z^{-n}$$

$= 0$ when $-n-1 = 0$

$-n = 1$

$n = -1$

$$= \sum_{n=-\infty}^{-1} 3^n z^{-n}$$

let $k = -n$

$$= \sum_{k=\infty}^{1} 3^{-k} z^{k}$$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k z^k$$

let $m = k-1 \Rightarrow k = m+1$

$$= \sum_{m=0}^{\infty} \left(\frac{z}{3}\right)^{m+1} = \frac{z}{3} \sum_{m=0}^{\infty} \left(\frac{z}{3}\right)^m = \boxed{\frac{z}{3}\left(\frac{1}{1-\frac{z}{3}}\right) \quad \text{ROC: } -3 < z < 3}$$

(20 pts) **3.** The Laplace transform of the unit impulse response of a system is

$$H(s) = \frac{1}{s+2}, \mathrm{Re}(s) > -2.$$

Determine the response $y(t)$ of the system when the input is

$$x(t) = \begin{cases} e^{-3t} & t > 0 \\ 0 & t = 0 \\ -e^{3t} & t < 0 \end{cases}.$$

$$x(t) = e^{-3t}u(t) - e^{3t}u(-t) = e^{-3t}\left(u(t) - u(-t)\right)$$

$$\mathcal{L}(x(t)) = \begin{cases} \frac{1}{s+3}, & \mathrm{Re}(s) > -3 \\ \frac{1}{s+3}, & \mathrm{Re}(s) < -3 \\ \text{undefined} & \mathrm{Re}(s) = -3 \end{cases} \Rightarrow \begin{cases} \text{undefined}, & \mathrm{Re}(s) = -3 \\ \frac{1}{s+3}, & \text{else} \end{cases}$$

$$Y(s) = H(s)\,X(s)$$

$$Y(s) = \frac{1}{s+2} \cdot \frac{1}{s+3} = \frac{A}{s+2} + \frac{B}{s+3} = \frac{1}{s+2} - \frac{1}{s+3}$$

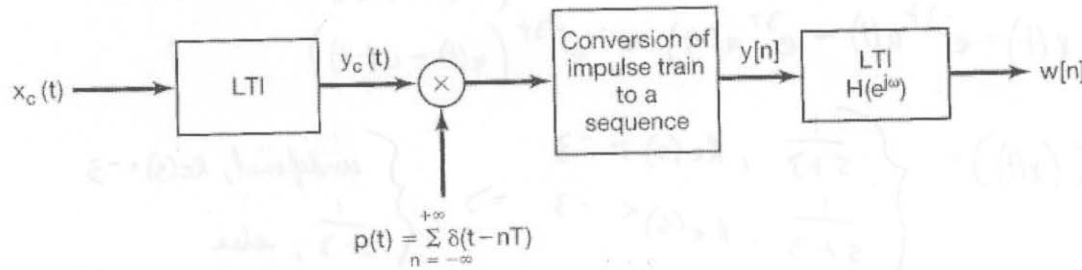
$$\left\downarrow \mathcal{L}^{-1}_{\mathrm{Re}(s) > -2} \right. \qquad \left\downarrow \mathcal{L}^{-1}_{\mathrm{Re}(s) \neq -3} \right.$$

$$\boxed{y(t) = e^{-2t}u(t) - e^{-3t}u(t)}$$

4. The block diagram below shows a system consisting of a continuous-time LTI system followed by a sampler, conversion to a sequence, and an LTI discrete-time system. The continuous-time LTI system is causal and satisfies the linear, constant-coefficient differential equation

$$\frac{dy_c(t)}{dt} + y_c(t) = x_c(t).$$

The input $x_c(t)$ is a unit impulse $\delta(t)$.



(10 pts) **a)** Determine the input $y_c(t)$.

$sY_c(s) + Y_c(s) = X(s)$, by (40) (46)

$Y_c(s) = \frac{X(s)}{s+1}$

$X(s) = 1$ by (54)

$Y_c(s) = \frac{1}{s+1}$

$\mathcal{L}^{-1}$ · causal systems are right sided ∴ L.T. is also right sided.

$\boxed{Y_c(t) = e^{-t}u(t) \text{ , by (52)}}$

(Problem 7 continues on the next page.)

(15 pts) **b)** Determine the frequency response $H(e^{j\omega})$ and the unit impulse response $h[n]$ such that $w[n] = \delta[n]$.

$$H'(\omega) = \frac{y(\omega)}{x(\omega)} = \frac{y(\omega)}{1}$$

to cancel that out so that $W(\omega) = 1$

$$H(\omega) = \frac{X(\omega)}{Y(\omega)}$$

$$y_c(t) = e^{-t} u(t)$$

$$y_c(nT) = e^{-nT} u(nT) = (e^{-T})^n u(nT)$$

$$y[n] = (e^{-T})^n u[n]$$

$$\xrightarrow{\mathcal{F}} \frac{1}{1-(e^{-T})e^{-j\omega}} \quad \text{when } |e^{-T}| < 1 \quad \text{by (38)}$$

$$y(\omega) = \frac{1}{1 - e^{-T}e^{-j\omega}}$$

$$H(\omega) = \frac{1}{\frac{1}{1-e^{-T}e^{-j\omega}}}$$

$$\boxed{H(\omega) = 1 - e^{-T}e^{-j\omega}}$$

$$\xrightarrow{\mathcal{F}^{-1}} = \delta[n] - e^{-T}\delta[n-1] \quad \text{by (36)(26)}$$

$$\boxed{h[n] = \delta[n] - e^{-T}\delta[n-1]}$$