Exponential r.v.s are used to model the time between occurrences of an event (arrivals, failures) and ~~on~~ the lifetime of devices.

In most situations, λ is in units event / (time unit) where the time unit is seconds, minutes, hours, etc.

$\Rightarrow$ This means $\frac{1}{\lambda}$ is in units (time unit) / event
Sometimean $1/\lambda$ is referred to as the average arrival time, average lifetime, etc...

The exponential r.v. satisfies the memoryless property

$$Pr(X > t_0 + t \mid X > t_0) = Pr(X > t)$$

The L.H.S gives the probability of having to wait at least t after waiting t_0 given one has already been waiting t_0. The R.H.S gives the probability of waiting at least t from the start. Thus the probability of waiting at least t is the same regardless of when you start.

Pf: $Pr(X > t_0 + t \mid X > t_0) = \dfrac{Pr(\{X > t_0 + t\} \cap \{X > t\})}{Pr(X > t_0)}$

$$= \frac{Pr(\{X > t_0 + t\})}{Pr(X > t_0)}$$

$$= \frac{e^{-\lambda(t_0 + t)}}{e^{-\lambda t_0}}$$

$$= e^{-\lambda t} = Pr(X > t)$$

The memory less property can also written as

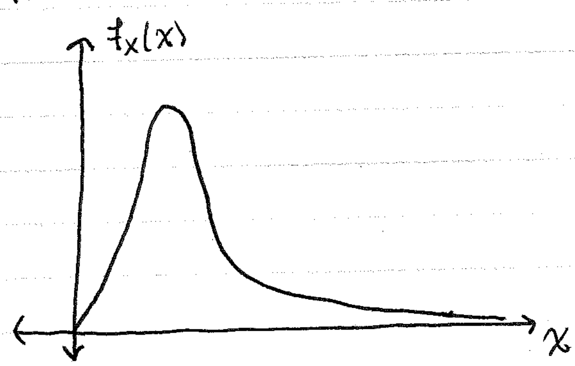
$$f_X(x \mid X > t_0) = f_X(x - t_0) \quad , t_0 > 0$$

Note: The exponential r.v. is the only continuous r.v. that satisfies the memory less property.

Rayleigh Distribution

The Rayleigh distribution is often found to model the amplitude of received communication signals.
A rayleigh r.v. X with parameter α has p.d.f.

$$f_X(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2} u(x) \quad , \quad \alpha > 0$$



$$E[X] = \alpha \sqrt{\pi/2}$$

$$Var[X] = (2 - \pi/2)\alpha^2$$

Important Discrete Random Variables

Bernoulli R.V.

Let A be an event. The Bernoulli r.v. is the value of 1_A (indicator function of A).
A Bernoulli r.v. with $Pr(A) = p$ has pmf

$$p_X(x_i) = \begin{cases} 1-p, & x_i = 0 \\ p, & x_i = 1 \end{cases}$$

$$E[X] = p \quad , \quad Var[X] = p(1-p)$$

Bernoulli random variables are useful in the anylyses of sequences of independent random experiments.

Binomial R.V.

Used to model the number of occurrences of an event A in n independent trials of a random experiment. A binomial r.v. X with $Pr(A) = p$ has pmf

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad , \quad k = 0, 1, \ldots, n$$

$$E[X] = np \quad , \quad Var[X] = np(1-p)$$

Geometric R.V.

Used to model the number of trials (or failures) until the occurrence of an event A, in independent trials of a random experiment.

A geometric r.v. X with $Pr(A) = p$ has pmf

1^{st} version (trials)

$$P_X(m) = (1-p)^{m-1} p \quad , \quad m = 1, 2, \ldots$$

$$E[X] = {}^{1-p}/_{p} \qquad Var[X] = 1-p/p^2$$

2^{nd} version (failures)

$$P_X(m) = (1-p)^{m} p \quad , \quad m = 0, 1, \ldots$$

$$E[X] = {}^{1}/_{p} \qquad Var[X] = 1-p/p^2$$

The geometric r.v. satisfies the memory less property

$$Pr(X \geq m_0 + m \mid X > m_0) = Pr(X \geq m)$$

Interpretation is similar to that of the exponential r.v., i.e., the probability of ~~to~~ having to perform at least m trials is the same regardless of how many trials have already been performed.

Pf: Left as exercise

Uniform R.V.

Used to model equally likely outcomes.
A uniform r.v. X on the set $S_X = \{x_1, x_2, \dots, x_n\}$ has pmf

$$p_X(x_i) = \frac{1}{n}, \qquad x_i \in \{x_1, x_2, \dots, x_n\}$$

If $S_X = \{1, 2, \dots, n\}$

$$E[X] = \frac{n+1}{2}, \qquad Var[X] = \frac{n^2-1}{12}$$

Characteristic Function

Transform methods are useful aids in the solution of equations with derivative and integrals.
We will discuss a Fourier-like transform of pdfs. This method will become very useful when discussing sums of random variables.

The characteristic function of r.v. X is defined as

$$\varphi_X(w) = E\left[e^{jwX}\right], \quad j = \sqrt{-1}$$

$$= \int_{-\infty}^{\infty} f_X(x)\, e^{jwx}\, dx$$

$\varphi_X(w)$ can be viewed as the expected value of e^{jwx} or * as the "Fourier" transform of $f_X(x)$. We can find $f_X(x)$ from $\varphi_X(w)$ as

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_X(w)\, e^{-jwx}\, dx$$