1. Let E_1 and E_2 be measurable, and show $|E_1 \cup E_2| + |E_1 \cap E_2| = |E_1| + |E_2|$.

2. (a) Let $\{q_1, q_2, \ldots\}$ be an enumeration of the set of rational numbers in $(0,1)$. Define $f : [0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2^{-n} & x = q_n \\ 0 & \text{otherwise} \end{cases}.$$

   Is f of bounded variation? Why?

   (b) Give an example of a function $f : [0,1] \to \mathbb{R}$ such that $f = 0$ almost everywhere and f does not have a bounded variation. Justify your answer.

3. Let $f_n : \mathbb{R} \to \mathbb{R}$, measurable with $f_n \geq f_{n+1}$ and $f_n \to f$ pointwise. Suppose also there exists $g \in L^1$ with $g \geq f_n$. Show $\lim_{n\to\infty} \int f_n = \int f$.

4. (53) Let $f_n \in L^p([0,1]), 1 < p < \infty$ with $||f_n||_p \leq 1$ for all n. Set $F_n(x) = \int_0^x f_n(t)dt$. Prove that F_n has a subsequence which converges uniformly on $[0,1]$.

5. (I-4) Suppose $f : [0,1] \to \mathbb{R}$ is differentiable at every $x \in [0,1]$ where by differentiable at 0 and 1, we understand that as left and right differentiability, respectively. Prove that f' is continuous if and only if f is uniformly differentiable, i.e, if and only if for all $\epsilon > 0$, there is an $h_0 > 0$ such that

$$\left| \frac{f(x+h) - f(x)}{h} - f'(x) \right| < \epsilon$$

   whenever $0 \leq x, x+h \leq 1$ and $0 < |h| < h_0$.