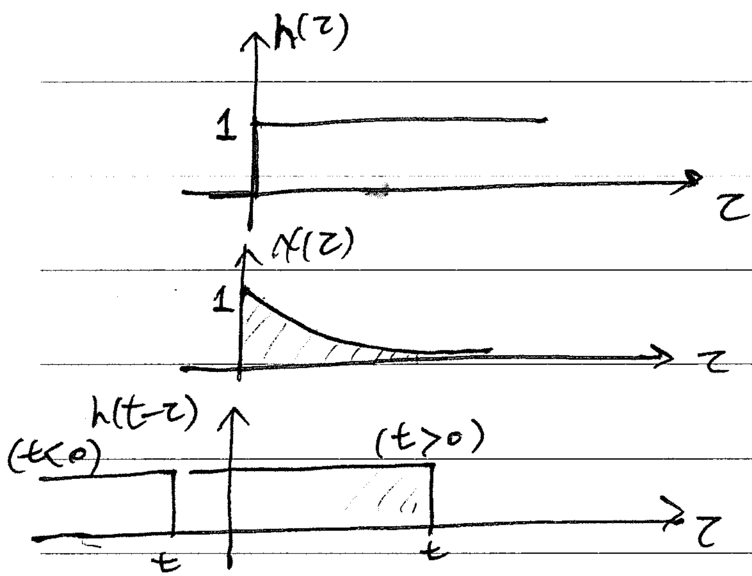
Example 2.6 (p.98–99)

$$\text{“} \underbrace{e^{-at}u(t)}_{=x(t)} * \underbrace{u(t)}_{=h(t)} = \frac{1}{a}\left(1-e^{-at}\right)u(t) \text{”}$$

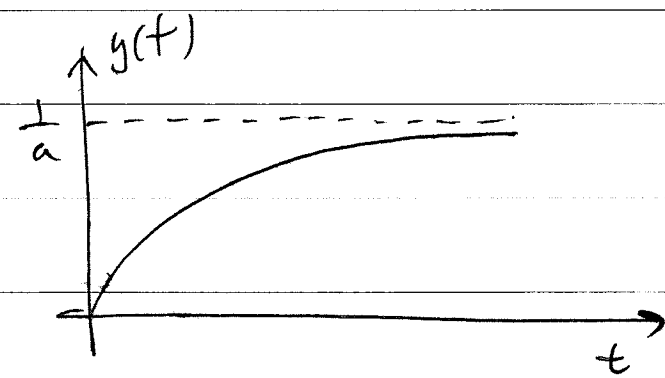
$$\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$



$$y(t) = \int_0^t e^{-a\tau}\,d\tau = -\frac{1}{a}e^{-a\tau}\Big|_0^t$$

$$= \begin{cases} \frac{1}{a}\left(1-e^{-at}\right), & t>0 \\ 0, & t<0 \end{cases}$$

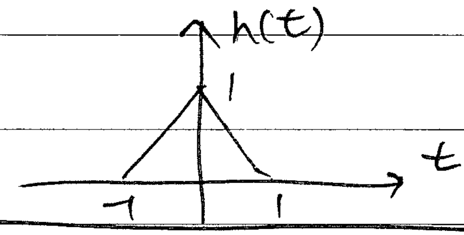
$$= \frac{1}{a}\left(1-e^{-at}\right)u(t)$$



Prob. 2.23 (pg. 143)

$y(t) = x(t) * h(t)$

, where $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$ &



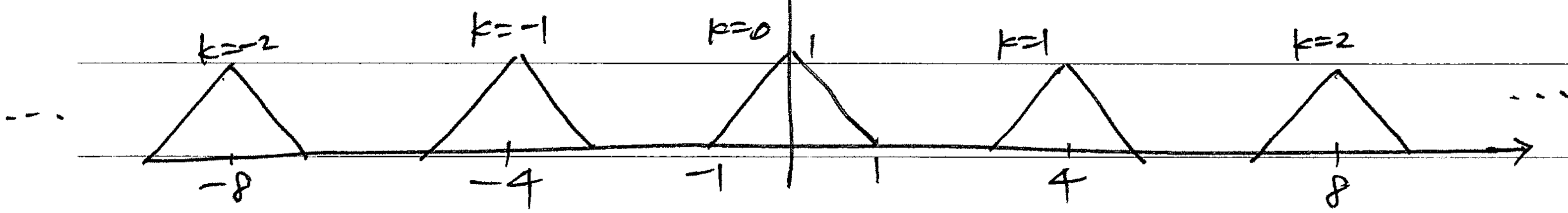
$\rightarrow y(t) = \left\{ \sum_{k=-\infty}^{\infty} \delta(t-kT) \right\} * h(t)$

$= \sum_{k=-\infty}^{\infty} h(t-kT)$

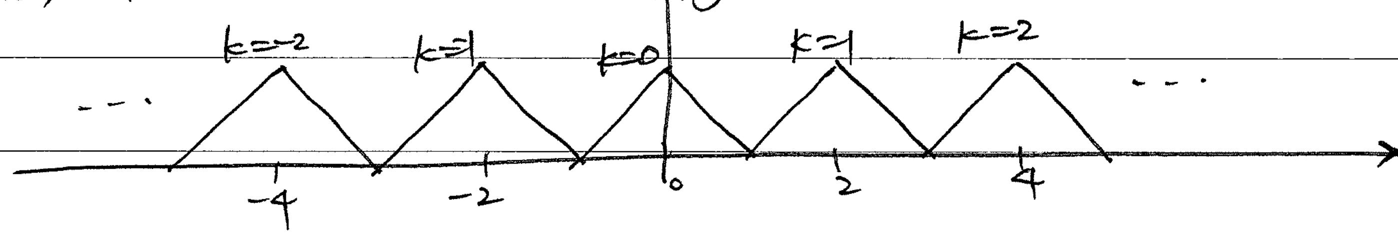
* hint : " $h(t) * \delta(t-kT) = h(t-kT)$ "

plot y(t) for T = 4 , 2 , 1.5 , 1

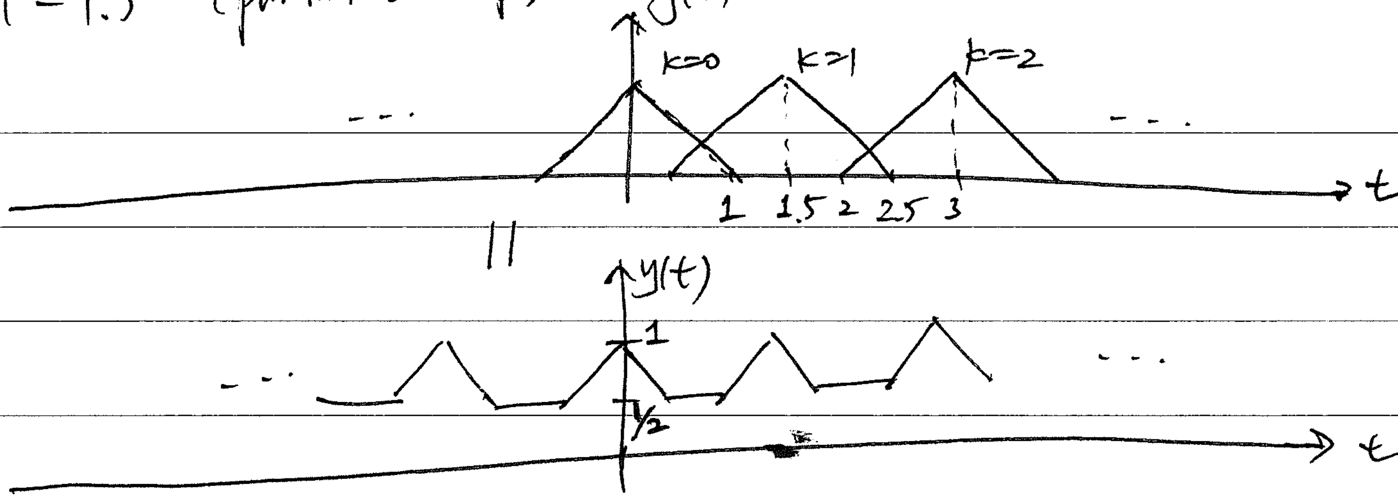
(a) T = 4 (no overlap)



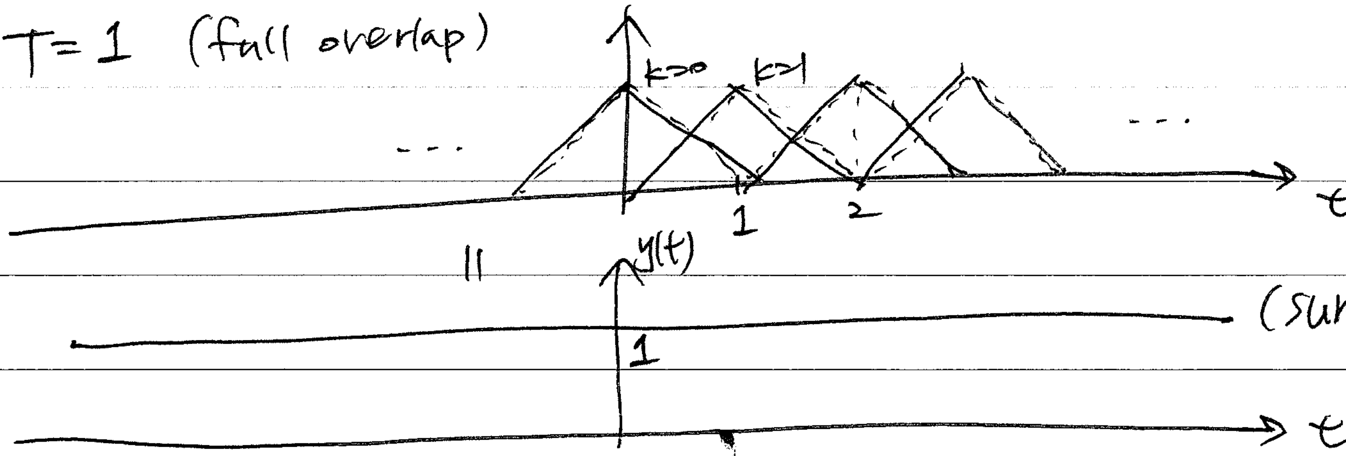
(b) T = 2



(c) $T = 1.5$ (partial overlap)



(d) $T = 1$ (full overlap)



(sum=1 for all k)

Prob. 2.40 $$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau$$

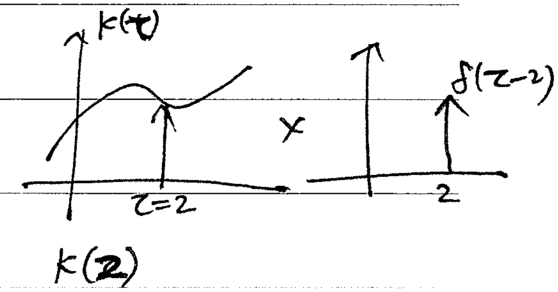
(Impulse Response)

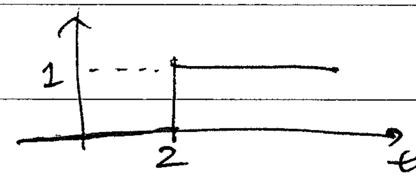
- $h(t)=?$ $x(t)=\delta(t)$, $x(\tau-2)=\delta(\tau-2)$

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau$$

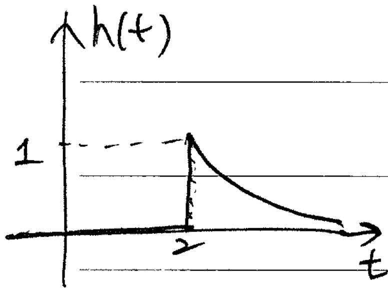
$$= \int_{-\infty}^{t} \underline{e^{-(t-2)}} \delta(\tau-2)\, d\tau$$

$$= e^{-(t-2)} \cdot \underline{\int_{-\infty}^{t} \delta(\tau-2)\, d\tau} = \begin{cases} 1 & \text{if } t>2 \\ 0 & \text{o.w} \end{cases}$$



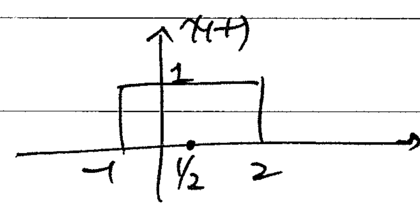


$= u(t-2)$



$$= \underline{e^{-(t-2)} u(t-2)}$$

- Find $y(t)$ when $x(t) = u(t+1) - u(t-2) = \text{rect}\left(\frac{t-1/2}{3}\right)$



(Recall: $e^{-at}u(t) * u(t) = \frac{1}{a}(1-e^{-at})u(t)$

This is a special case of $\underline{e^{-\alpha t}u(t) * e^{-\beta t}u(t)}$ with $\beta = 0$)

- Thus: $\{u(t) - u(t-T)\} * e^{-at}u(t)$ (distributive —)

$$= \frac{1}{a}(1-e^{-at})u(t) - \frac{1}{a}(1-e^{-a(t-T)})u(t-T)$$

$$= \frac{1}{a}(u(t)-u(t-T)) - \frac{1}{a}e^{-at}(u(t)-e^{aT}u(t-T))$$

use

(Recall: If $y(t) = x(t) * h(t)$

Then $x(t-t_1) * h(t-t_2) = y(t-(t_1+t_2))$)

- $x(t)$ is $\{u(t)-u(t-3)\}$ shifted to the right by $t_1 = -1$

e^{-t} $h(t)$ is $e^{-2t}u(t)$ shifted to the right by $t_2 = 2$

$a=1$ with $a=2$ and $T=3$: $(u(t)-u(t-3)) * e^{-2t}u(t)$ is

$$\frac{1}{2}(u(t)-u(t-3)) - \frac{1}{2}e^{-2t}(u(t)-e^{2\cdot 3}u(t-3))$$

- Thus answer is above shifted to the right by

$t_1 + t_2 = -1 + 2 = 1$

- Answer: $y(t) = \frac{1}{2}(u(t-1)-u(t-4)) - e^{-2(t-1)}(u(t-1)-e^{2\cdot 3}u(t-4))$