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Instructions: This is an open notes, open book, collaborative quiz. No Internet allowed.

**Question:** Let  $X$  be a discrete random variable with probability mass function

$$p_X(k) = \frac{C}{3^{|k|}}, \quad k \in \mathbb{Z},$$

for an appropriately chosen constant  $C$ . What is the expectation of  $X$ ?

**Answer:**

$$E(X) = \sum_{k=-\infty}^{\infty} k p_X(k) = \sum_{k=-\infty}^{\infty} k \frac{C}{3^{|k|}} = \sum_{k=-\infty}^0 k \frac{C}{3^{-k}} + \sum_{k=1}^{\infty} k \frac{C}{3^k}$$

$$= \sum_{r=0}^{\infty} \overset{\text{letting } r=-k}{-r} \frac{C}{3^r} + \sum_{k=1}^{\infty} k \frac{C}{3^k} \quad \text{since } k=0 \text{ term is zero.}$$

$$= C \left( \sum_{k=0}^{\infty} k \frac{C}{3^k} - \sum_{r=0}^{\infty} r \frac{C}{3^r} \right) = 0$$

Note: if you had to find  $C$ , use axiom II

$$\sum_k p_X(k) = 1$$

↳ Rhea problem

$$\Rightarrow \sum_k \frac{C}{3^{|k|}} = 1$$

$$\Rightarrow \sum_{k=-\infty}^0 \frac{C}{3^{-k}} + \sum_{k=1}^{\infty} \frac{C}{3^k} = 1$$

$$\Rightarrow \sum_{r=0}^{\infty} \frac{C}{3^r} + \sum_{k=0}^{\infty} \frac{C}{3^k} - C = 1$$

Score: / 10

$$\Rightarrow \frac{C}{1 - \frac{1}{3}} + \frac{C}{1 - \frac{1}{3}} - C = 1$$

$$\Rightarrow C \left( \frac{3}{2} + \frac{3}{2} - 1 \right) = 1 \quad \Rightarrow C = \frac{1}{2}$$