

# 3.6 DT Fourier Series (Periodic Signals)

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3:07 PM

$$z^n \rightarrow (\sqrt{2}j)^n \rightarrow \boxed{\text{LTI}} \rightarrow H(\sqrt{2}j) \cdot (\sqrt{2}j)^n$$

$$\sum a_k e^{jk\omega_0 n} \rightarrow \boxed{\text{LTI}} \rightarrow \sum a_k H(e^{jk\omega_0}) e^{jk\omega_0 n}$$

Use Fourier to write signals as summations of complex exponentials

## Fourier Series Formula

Let  $x[n]$  be a periodic DT signal with fundamental period  $N$

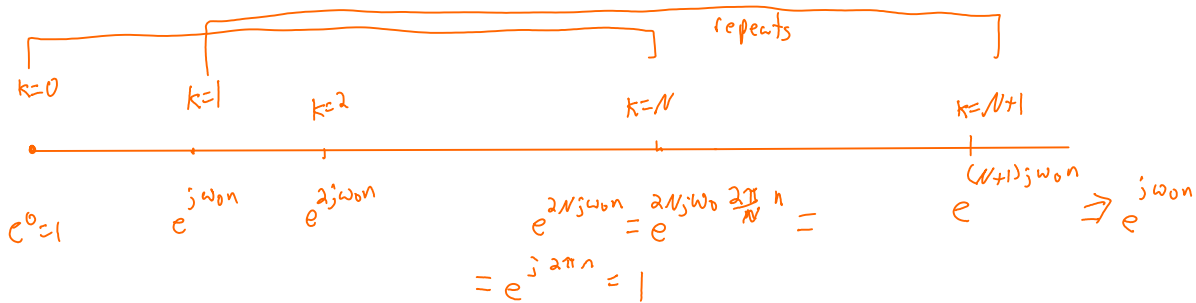
Write  $\omega_0 = \frac{2\pi}{N}$

Then  $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n}$ , where  $a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n}$

why a finite number in sum?

Because the set  $\{e^{jk\omega_0 n}\}_{k \in \mathbb{Z}}$  (harmonically related exponentials)

contains a finite number of distinct functions.



it repeats so you only need to go from  $0 \rightarrow N-1$  to get all values.

Question: Obtain the DT Fourier series coefficients  $(a_k)$  of the signal

$$x[n] = \sin(3\pi n + \frac{\pi}{2})$$

$$N = \frac{2\pi}{3\pi} \cdot k = \frac{2}{3}k, \text{ take } k=3 \text{ then } N=2 \text{ *important to know}$$

Note: the function  $x[n] = (-1)^n$  is same  $\sin(3\pi n + \frac{\pi}{2})$  in DT

$$\begin{aligned} x[0] &= 1 \\ x[1] &= -1 \\ x[2] &= 1 \\ x[3] &= -1 \end{aligned}$$

$$a_k = \frac{1}{2} \sum_{n=0}^1 x[n] e^{-jk\pi n}$$

$$a_0 = \frac{1}{2} \sum_{n=0}^1 x[n] e^0 = 0$$

$$a_1 = \frac{1}{2} 1e^0 + \frac{1}{2} (-1)e^{-j\pi} = \frac{1}{2} + \frac{1}{2} e^{-j\pi} = \frac{1}{2} - \frac{1}{2}(-1) = 1$$

$$x[n] = 0 \cdot e^0 + 1e^{j\pi n} = e^{j\pi n} \text{ which is same as } \sin(3\pi n + \frac{\pi}{2})$$

Another way to get  $a_k$ 's

$$x[n] = \sin(3\pi n + \frac{\pi}{2}) \stackrel{\text{Euler's}}{=} \frac{e^{j(3\pi n + \frac{\pi}{2})} - e^{-j(3\pi n + \frac{\pi}{2})}}{2j}$$

split

$$= \frac{1}{2j} e^{j\frac{\pi}{2}} e^{j3\pi n} - \frac{1}{2j} e^{-j\frac{\pi}{2}} e^{-j3\pi n}$$

$$= \frac{1}{2} e^{j3\pi n} + \frac{1}{2} e^{-j3\pi n}$$

$\xrightarrow{\text{add } -2\pi \text{ (one period)}} \quad \xrightarrow{\text{add } 4\pi \text{ (2 periods)}}$

$$= \frac{1}{2} e^{j5\pi n} + \frac{1}{2} e^{j\pi n}$$

because  $e^{j\pi}$  is periodic with period  $2\pi$

$$= 1e^{j5\pi n} \leftarrow k=1 \text{ so } a_{k=1}$$

No  $k=0$  term  
so  $a_k=0$