DT Fourier

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3:02 PM

$$\underset{DT}{x[n]} \xrightarrow{F} X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$\mathcal{F}^{-1}(X(\omega)) = \frac{1}{2\pi}\int_0^{2\pi} X(\omega)\, e^{j\omega n}\, d\omega = x[n]$$

Ex. 1

$$x[n] = 2^{-n} u[n]$$

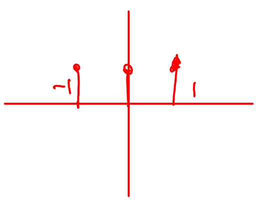
$$X(\omega) = \sum_{n=-\infty}^{\infty} 2^{-n} u[n]\, e^{-j\omega n} = \sum_{n=0}^{\infty} 2^{-n} e^{-j\omega n} =$$

$$\left| \frac{e^{-j\omega n}}{2^n} \right| < 1$$

$$= \frac{1}{1 - \frac{e^{-j\omega}}{2}} \quad \text{(geometric)}$$

Exercise

$$x[n] = u[n+1] - u[n-2]$$



$$X(\omega) = \sum_{n=-\infty}^{\infty} u[n+1] - u[n-2] = \sum_{n=-1}^{1} \overset{e^{-j\omega n}}{e^{-j\omega n}} =$$

$$= e^{j\omega} + 1 + e^{-j\omega} = 1 + 2\cos(\omega)$$

Properties of DTFT:

1) Periodic with 2π

$$X(\omega) = \sum x[n]\, e^{-j\omega n}$$

$$X(\omega + 2\pi \ell) = \sum x[n]\, e^{-j\omega n}\, \overset{1}{\cancel{e^{-j2\pi \ell n}}} = X(\omega)$$

2) linearity

$$\mathcal{F}(\alpha x[n] + \beta y[n]) = \alpha X(\omega) + \beta Y(\omega)$$

ex. $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$

$$X(\omega) = \sum_{k=0}^{N-1} a_k \mathcal{F}\left(e^{j k \frac{2\pi}{N} n}\right)$$

$0 < \omega < 2\pi$

since $e^{j\omega_0 n} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega-\omega_0)$ - repeating delta function

$$\text{so } X(\omega) = \sum_{k=0}^{N-1} a_k \mathcal{F}\left(e^{\frac{jk2\pi}{N} n}\right) = \sum_{k=0}^{N-1} a_k \underbrace{\sum_{\ell=-\infty}^{\infty} 2\pi\delta\left(\omega - \frac{k2\pi}{N} - 2\pi\ell\right)}_{\text{repeating delta part}} =$$

$$= \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$$

$$\mathcal{F}^{-1}\left(\sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right)\right) = \cancel{\frac{1}{2\pi}}\int_0^{2\pi} \sum_{k=-\infty}^{\infty} \cancel{2\pi} a_k \delta\left(\omega - \frac{2\pi k}{N}\right) e^{j\omega n} d\omega =$$

$$= \sum_{k=0}^{N-1} \int_0^{2\pi} a_k \delta\left(\omega - \frac{2\pi k}{N}\right) e^{j\omega n} d\omega = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi k}{N} n}$$

3) Time shifting & frequency shifting

$$\mathcal{F}(x[n-n_0]) = e^{-j\omega n_0} X(\omega)$$

$$\mathcal{F}(e^{j\omega_0 n} x[n]) = X(\omega-\omega_0)$$

4) Conjugation

$$\mathcal{F}(x^*[n]) = X^*(-\omega)$$

if $x[n]$ is real:

$$x[n] \xrightarrow{\mathcal{F}} X(\omega) = X^*(\omega)$$

$$X(\omega) = Re(\omega) + j\,Im(\omega)$$

$$X^*(\omega) = Re(\omega) - j\,Im(\omega)$$

so Real part of $X(\omega)$ is even

$$Re(\omega) = Re(-\omega)$$

and Imaginary part of $X(\omega)$ is odd

$$Im(\omega) = -Im(-\omega)$$

5) Parseval's Relation

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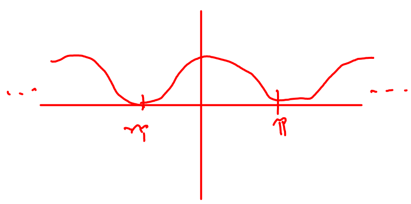
$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_0^{2\pi} |X(\omega)|^2 \, d\omega$$

6) Convolution

$$x[n] * y[n] \xrightarrow{\mathcal{F}} X(\omega)Y(\omega)$$

7) There is No Duality in DTFT

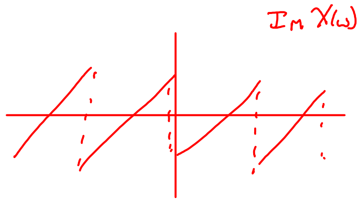
Ex 2.



$x[n]$

is $x[n]$ periodic, No because if it was $X(\omega)$ would be a train of deltas.

$Im\, X(\omega)$



is $x[n]$ real & even

-not even because if it was there wouldn't be any Im part

it is real why?

Finite energy?