1. Let C be the Cantor set. For all $x \in [0,1)$, let $.x_1x_2x_3...$ be the ternary decimal expansion for x, and furthermore, if $x = \frac{k}{3^n}$ we require only finitely many nonzero elements of the expansion to ensure uniqueness. Let $\mathcal{A}$ be the sigma-algebra generated by the sets $\{x : \exists \text{ i with } x_i = 1\}$. Prove or disprove $C \in \mathcal{A}$.

2. Suppose $|g_n(x)| \leq M$ for all $n \geq 1$ and all $x \in [0,1]$ and that
$$\int_a^b g_n(x)dx \to 0$$
whenever $0 \leq a \leq b \leq 1$. Show that
$$\int_0^1 f(x)g_n(x)dx \to 0$$
for all $f \in L^1([0,1])$.

3. Let $(X, \mathcal{M}, \mu)$ be a finite measure space. Assume that $f_n \to f$ μ-a.e. and
$$\sup_n \int_X |f_n|^{p_0} < \infty$$
for some $1 < p_0 < \infty$. Show that $f_n \to f$ in L^1.

4. Let μ be a measure on $[0,1]$ by $\mu(\{0\}) = \mu(\{1\}) = 1$, and for all $(a,b) \subset [0,1], \mu(a,b) = b\ln(b) - a\ln(a) - b + a$. Let $f \in C[0,1]$ and show
   (a) $\int_a^b f d\mu = \int_a^b f(x)\ln(x)dx + f(0)\chi_{(a,b)}(0) + f(1)\chi_{(a,b)}(1)$.
   (b) $f \in L^1(\mu)$.

5. Let $f \in L^1([0,1])$ and assume that
$$\int_0^1 \frac{|f(t)|}{(1-t)^2}dt < \infty.$$
Let $f_n(x) = f(x^{1/n})$. Show that $\sum_{n=1}^{\infty} f_n(x)$ converges absolutely a.e. $x \in [0,1]$.