EE 301 Exam 1 9/15/04

Name: ____________________

General Instructions:

- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- [redacted]
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 100 points.
- Do all work in the blue books provided. Put your name and student identification number on the blue book. This exam problem sheet **must be handed in** with your blue book.
- All plots must be carefully draw with axes labeled.
- If you finish the exam during the first 50 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up your exam and blue book from you.
- This exam is for Krogmeier's section only.

Do not open the exam until you are told to begin.

1. (30 pts.) Give answers and short explanations for each of the following unrelated problems:

   (a) The discrete-time complex exponential signal $x[n] = e^{j\omega_0 n}$ is input to a discrete-time LTI system with impulse response $h[n]$. Show that the output of the system is of the form

   $$y[n] = e^{j\omega_0 n} H(\omega_0)$$

   where $H(\omega_0)$ is a complex number independent of time n. Find a formula for $H(\omega_0)$.

   (b) Let $x(t)$ be a continuous-time signal and let $y(t) = x(at + b)$. Find a formula which relates $E_\infty(x)$, the energy in x, to $E_\infty(y)$, the energy in y.

   (c) For each of the systems defined below, decide if it is linear, time-invariant, memoryless, causal, and/or stable. In each case, the input is denoted by x and the output by y. Explain.

      i. $y[n] = x[n] + 2x[n-1]$.
      ii. $y[n] = x[n] + 0.5x[n+1]$.
      iii. $y[n] = \sum_{k=n-2}^{n+2} kx[k]$.
      iv. $y[n] = e^{x[n]}$.

   (d) If

   $$x(t) = \begin{cases} t & \text{for } 0 \le t \le 1 \\ 0 & \text{for } t < 0 \text{ or } t > 1 \end{cases}$$

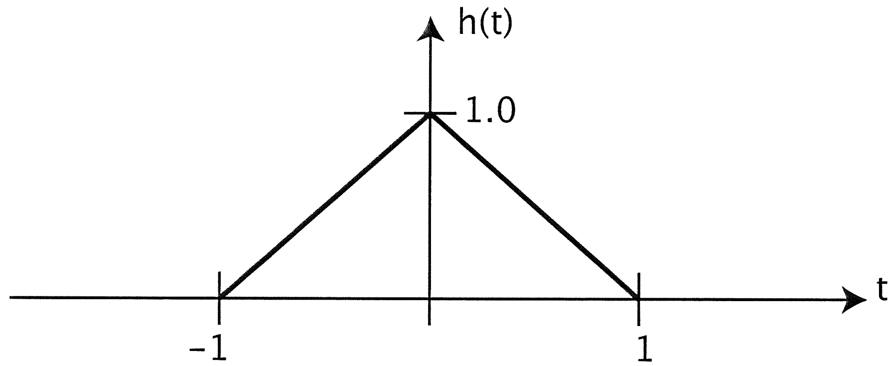
   plot the signal $y(t) = x(3 - 0.5t)$.

   (e) Let $h[n] = (\frac{1}{2})^n u[n]$ be the impulse response of an LTI system in discrete-time. A causal inverse system exists whose impulse response $h_{inverse}[n]$ is nonzero only for $n = 0$ and $n = 1$. Find the values $h_{inverse}[0]$ and $h_{inverse}[1]$.

2. (20 pts.) Solve the following differential equation for $y(t)$, $t \ge 0$, assuming initial conditions $y(0) = 1$ and $y^{(1)}(0) = 0$:

   $$y^{(2)} + y = e^t.$$

3. (50 pts.) Let a continuous-time LTI system have the triangularly shaped impulse reponse $h(t)$ shown below.



(a) If the input to the system above is the rectangular signal

$$x_a(t) = \begin{cases} 0 & \text{for } t < 0 \text{ and } t > 1 \\ 1 & \text{for } 0 \leq t \leq 1 \end{cases},$$

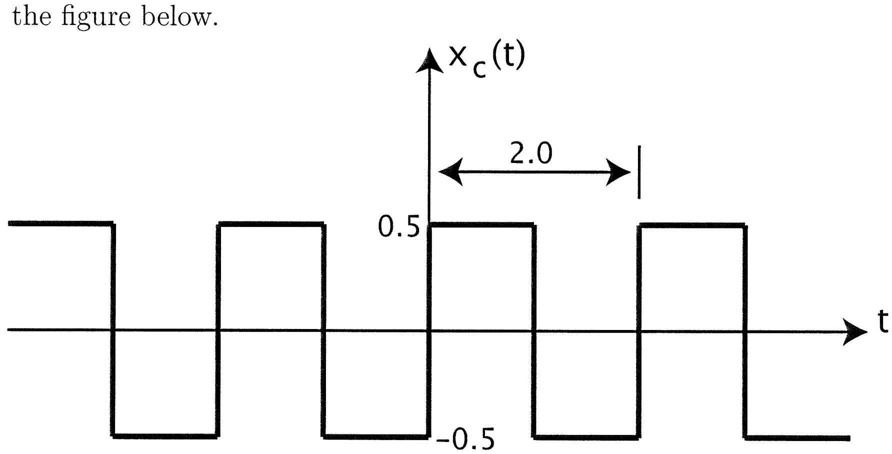
compute the output $y_a(t)$ and plot it.

(b) Now suppose the input is changed to

$$x_b(t) = x_a(t) + x_a(t-2).$$

Compute the corresponding output $y_b(t)$ and plot. Use properties of LTI systems to avoid unnecessary work.

(c) Finally suppose the input is changed to the periodic square wave $x_c(t)$ shown in the figure below.



Compute and plot the corresponding output $y_c(t)$ again avoiding unnecessary work.