Midterm Examination 1
ECE 438
Fall 2010
Instructor: Prof. Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete the 5 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. This exam contains 11 pages. Page 8 contains a table of formulas. Page 9 contains the cheat sheet that was created by students on Rhea. Pages 10-11 are scratch paper. You may tear out the table and the scratch paper **once the exam begins**.
4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden. Ipods and PDAs are not allowed either.

Name:______________________________________

Email:______________________________________

Signature:____________________________________

Itemized Scores

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

(30 pts) **1.** Consider the continuous-time signal $x(t) = e^{j2\pi 880t}$ and the discrete-time signal $y[n] = x(0.001n)$.
a) What is the CTFT of $x(t)$? (Justify your answer.)

b) What is the DTFT of $y[n]$? (Give a mathematical expression, and sketch it.)

(20 pts) **2.** Compute the z-transform of $x[n] = \frac{1}{3^n}u[n-1] + 5^n u[-n]$.

(15 pts) **3.** Obtain the inverse z-transform of

$$X(z) = \frac{3}{z^2} + 8, \text{ for all } z \in \mathbb{C} \text{ such that } z \neq 0.$$

(Note: you do *not* need to express your answer using the unit step function.)

(20 pts) **4.** Compute the DFT of the discrete-time signal $x[n] = j^n$.

(35 pts) **5.** Consider a continuous-time signal $x(t)$ and three different samplings of this signal, namely

$$
\begin{aligned}
x_1[n] &= x(n\Delta t), \\
x_2[n] &= x(nD\Delta t), \\
x_3[n] &= x\left(\frac{n\Delta t}{D}\right),
\end{aligned}
$$

where D is a positive integer, and Δt is a positive real number.
a) Is it possible to reconstruct $x_2[n]$ from $x_1[n]$ (without explicitely reconstructing $x(t)$)? If so, explain how. If not, explain why.

Problem 5, continued.

b) Is it possible to reconstruct $x_3[n]$ from $x_1[n]$ (without explicitely reconstructing $x(t)$)? If so, explain how. If not, explain why.

Table

CT Fourier Transform

$$\text{F.T. : } \mathcal{X}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \quad (1)$$

$$\text{Inverse F.T.: } x(t) = \int_{-\infty}^{\infty} \mathcal{X}(f)e^{j2\pi t}df \quad (2)$$

DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad (3)$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \quad (4)$$

Discrete Fourier Transform

Let $x[n]$ be a periodic discrete-time signal with period N

$$\text{D.F.T.: } X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} \quad (5)$$

$$\text{Inverse D.F.T.: } x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} \quad (6)$$

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (7)$$

Student-Created Cheat Sheet

This is the cheat-sheet that was created by students on Rhea. I do not guarantee that it is completely free of mistakes, but it does not contain anything that would prevent you from being able to solve the exam correctly.

Work in progress for a formula sheet add things on :P?

- Fourier series of a continuous-time signal x(t) periodic with period T
- Fourier series coefficients of a continuous-time signal x(t) periodic with period T

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j\frac{2\pi}{T}nt} \qquad a_n = \frac{1}{T}\int_0^T x(t)e^{-j\frac{2\pi}{T}nt}dt$$

CTFS

$$x(t) = \int_{-\infty}^{\infty} \chi(f)\ e^{j2\pi ft}\,df \qquad \chi(f) = \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi ft}\,dt$$

CTFT

$$X[k] = \sum_{k=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}}$$

DFT

$$x[n] = \frac{1}{N}\sum_{n=0}^{N-1} X[k]e^{j\frac{2\pi kn}{N}}$$

IDFT

$$rep_T[x(t)] = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-kT) \qquad comb_T[x(t)] = x(t).\sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$rep_T[x(t)] \iff \frac{1}{T}comb_{\frac{1}{T}}[\mathrm{X}(f)] \qquad comb_T[x(t)] \iff \frac{1}{T}rep_{\frac{1}{T}}[\mathrm{X}(f)]$$

$$\delta(\alpha f) = \frac{1}{\alpha}\delta(f) \quad for\ \ \alpha > 0 \qquad sinc(\theta) = \frac{sin(\pi\theta)}{\pi\theta}$$

$$e^{j\pi} = -1 \qquad \cos(\theta) = \frac{(e^{j\theta} + e^{-j\theta})}{2} \qquad sin(\theta) = \frac{(e^{j\theta} - e^{-j\theta})}{2j}$$

$$\mathcal{F}(\frac{rect((t-\frac{T}{2})}{T})) \Rightarrow T sinc(Tf)(e^{-j2\pi f\frac{T}{2}})$$

Z-transform

$$Z(x[n]) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Z – transform

-SCRATCH -
(will not be graded)

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